EE16A

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OH Today 11-12

Reminders: HW self-grades to

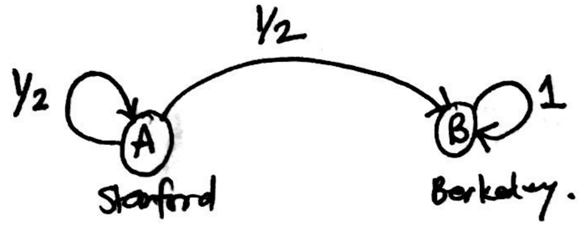
Lab schedule changed.

Extra Credit - Piazza.

Today:

- Page Rank
- Eigenvalue + Eigenspace.
- Determinant.

Two webpages.



$Q = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix}$

"Website B was the most popular"

$\vec{x}(t+1) = Q \cdot \vec{x}(t)$.

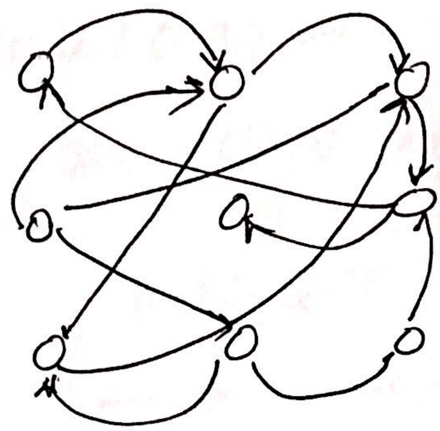
Time 1: $\vec{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\vec{x}(1) = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$, $\vec{x}(2) = \begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix}$, $\vec{x}(3) = \begin{bmatrix} 1/8 \\ 7/8 \end{bmatrix}$

$\vec{x}(t) = \begin{bmatrix} (1/2)^t \\ 1-(1/2)^t \end{bmatrix}$

$t \to \infty$ $\quad \vec{x}(\infty) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ "Steady State"

New: $\vec{x}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\vec{x}(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



$$\vec{x}(t+1) = \begin{bmatrix} & & \\ & Q & \\ & & \end{bmatrix} \vec{x}(t).$$

$t+1$

fractions of people at t

"transformation matrix"

Steady-state

$$\vec{x}(t+1) = Q \cdot \vec{x}(t) = 1 \cdot \vec{x}(t).$$

$$Q \cdot \vec{x}(t) - \vec{x}(t) = \vec{0}$$

$$Q \cdot \vec{x}(t) - I \cdot \vec{x}(t) = \vec{0}$$

$$(Q - I) \cdot \vec{x}(t) = \vec{0}$$

Find Null$(Q-I)$

$$Q = \begin{bmatrix} 1/2 & 0 \\ 1/2 & 1 \end{bmatrix} \qquad Q - I = \begin{bmatrix} -1/2 & 0 \\ 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1/2 & 0 \\ 1/2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-\frac{1}{2} x_1 + 0 \cdot x_2 = 0$$

$$\rightarrow \left[\begin{array}{cc|c} -1/2 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$x_2 = t$

$$\vec{x}(t) = \begin{bmatrix} 0 \\ 313 \end{bmatrix}$$

$$\text{span}\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}.$$

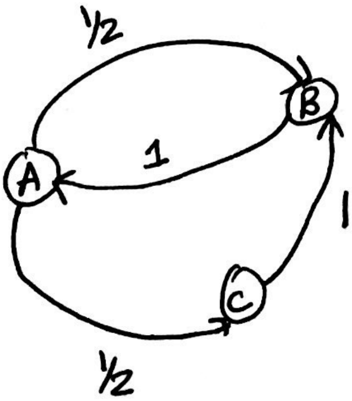
$$\vec{x}(t) = \alpha \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Null$(Q-I)$ = subspace spanned by $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\mathrm{Null}(Q-I) \to$ can you ever leave?

$\cancel{x}$ $x(t) \in \mathrm{Null}(Q-I)$.

$\mathrm{Null}(Q-I)$: eigenspace of Q.



$$Q = \begin{array}{c} \begin{array}{ccc} \vec{q}_1 & & \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1 \\ 1/2 & 0 & 0 \end{bmatrix} \end{array}$$

$$\vec{x}(t) = Q^t \cdot \vec{x}(0).$$

$$\lim_{t\to\infty} Q^t = \begin{bmatrix} .4 & .4 & .4 \\ .4 & .4 & .4 \\ .2 & .2 & .2 \end{bmatrix}.$$

$$Q \cdot \begin{bmatrix} .4 \\ .4 \\ .2 \end{bmatrix} = \begin{bmatrix} .4 \\ .4 \\ .2 \end{bmatrix}$$

What happens to

$\vec{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\quad Q \cdot \vec{e}_1 = \vec{q}_1$

$\vec{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\vec{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

elementary vectors.

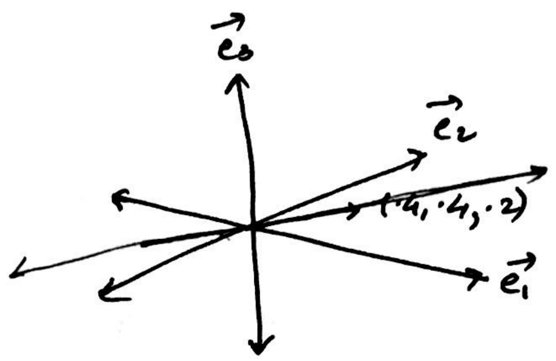
Basis for $\mathbb{R}^3$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = a\vec{e}_1 + b\vec{e}_2 + c\vec{e}_3$$

Eigenvalue

eigenvalue.

$$A\vec{v} = \lambda \cdot \vec{v}$$



$$A \cdot \vec{v} - \lambda \cdot \vec{v} = \vec{0}$$

$$(A - \lambda \cdot I) \cdot \vec{v} = \vec{0}.$$

$\text{Null}(A\,\lambda I)$: Eigenspace associated with eigenvalue λ.

$$B = (A - \lambda I)$$

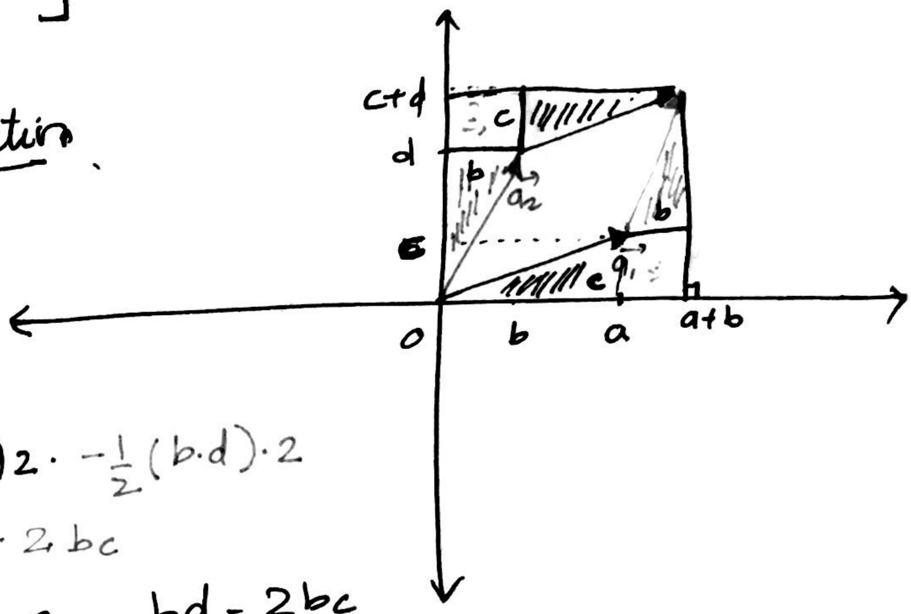
$$\begin{bmatrix} | & | & & | \\ \vec{b}_1 & \vec{b}_2 & \cdots & \vec{b}_n \\ | & | & & | \end{bmatrix} \vec{v} = \vec{0}$$

Find a λ such that $\vec{b}_1 \cdots \vec{b}_n$ are linearly dependent?

Determinant: Are the columns linearly dependent?

$$A = \begin{bmatrix} | & | \\ \vec{a}_1 & \vec{a}_2 \\ | & | \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Geometric interpretation.



$$(a+b)(c+d) - \left(\tfrac{1}{2}a\cdot c\right)2 \cdot - \tfrac{1}{2}(b\cdot d)\cdot 2$$
$$- 2bc$$

$$= ac + ad + bc + bd - ac - bd - 2bc$$

$$= ad - bc.$$

$$\text{Det}(A) = ad - bc$$

$$(A - \lambda I)\cdot \vec{v} = \vec{0}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \lambda I = \begin{bmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (a_{11}-\lambda)(a_{22}-\lambda) - a_{21}a_{12}$$

$$= \underbrace{\lambda^2 - \lambda(a_{11}+a_{22}) + a_{11}a_{22} - a_{21}a_{12}}_{\text{Characteristic polynomial.}} = 0.$$

$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

$A - \lambda I$ $\begin{bmatrix} 1-\lambda & 2 \\ 4 & 3-\lambda \end{bmatrix}$

Ch. Poly:

$(1-\lambda)(3-\lambda) - 2 \cdot 4 = 0$.

$\lambda^2 - 4\lambda - 5 = 0$.

$(\lambda+1)(\lambda-5) = 0$

Options $\lambda_1 = -1$, $\lambda_2 = 5$ Eigenvalues.

Eigenspace: $\text{Null}(A - (-1)I)$.

$\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} \vec{v} = \vec{0}$

$\text{Span}\left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$.

$\left[\begin{array}{cc|c} 2 & 2 & 0 \\ 4 & 4 & 0 \end{array}\right] \rightarrow$

$\left[\begin{array}{cc|c} 2 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$\text{Null}(A - 5I)$.

$\begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix} \vec{v} = \vec{0}$.

$\text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$.