

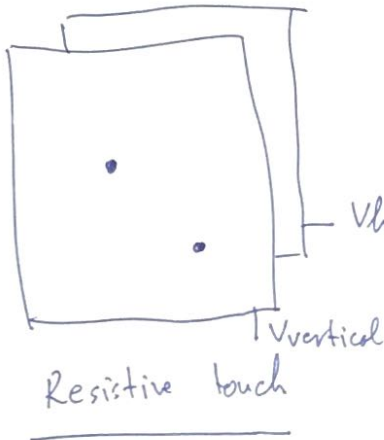
Lecture 6 - Module 2

Today:

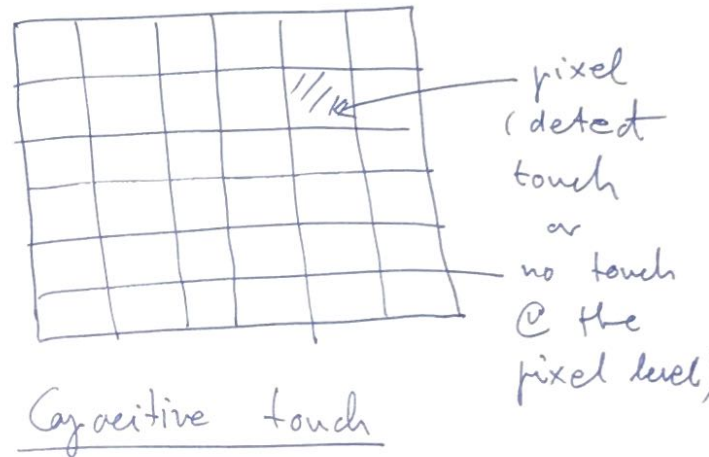
Note 16

- * Capacitive touchscreen
- * Capacitor equivalence
- * Capacitor physics

An improved touchscreen:

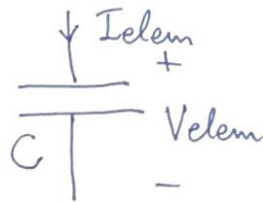


Arbitrary #
=> touch points



Circuit model:

Capacitor (symbol)



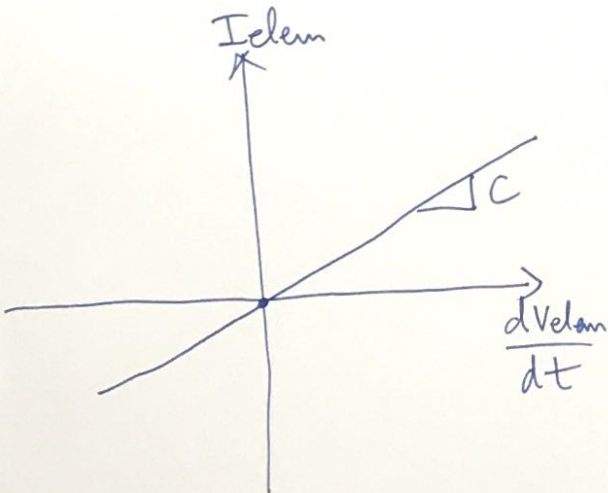
"I - V" element relationships

$$Q_{elem} = C \cdot V_{elem}$$

$[C] \uparrow$ $[F] \uparrow$ $[V] \uparrow$
 Farad

know: $I_{elem} = \frac{dQ_{elem}}{dt} = C \cdot \frac{dV_{elem}}{dt}$

C = const over time



$$I_{elem} = C \cdot \frac{dV_{elem}}{dt}$$

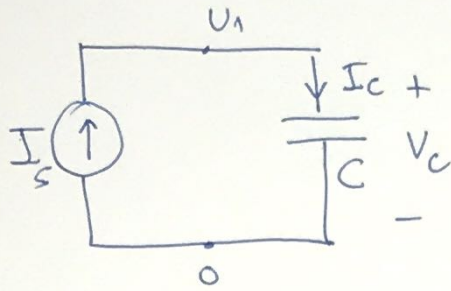
Can use the same

7-step dt analysis.

"Ohm's law" for a capacitor.

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Simple circuit # 1:



KCL: $I_s = I_c$

element def for C: $I_c = C \cdot \frac{dV_c}{dt}$

voltage def: $U_1 - 0 = V_c$

$$I_s \cdot dt = C dU_1$$

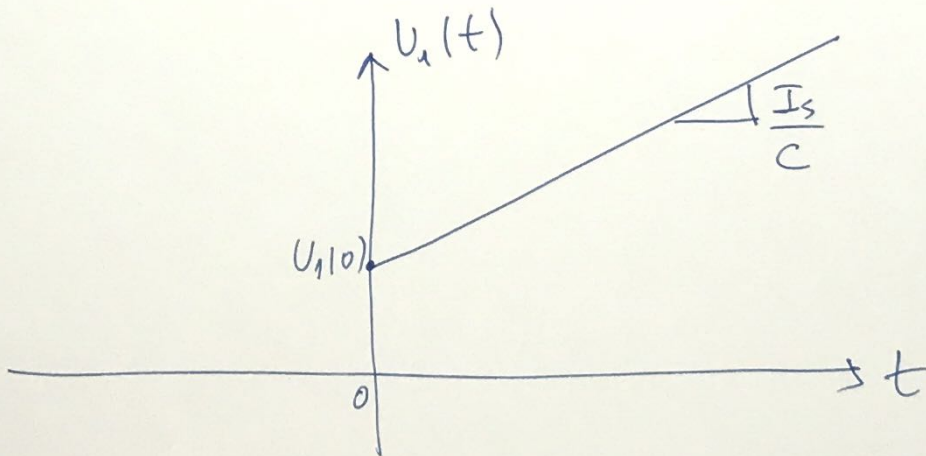
$$\int_0^t I_s dt = \int_{U_1(0)}^{U_1(t)} C dU_1$$

\Downarrow

$$I_s = C \frac{dU_1}{dt}$$

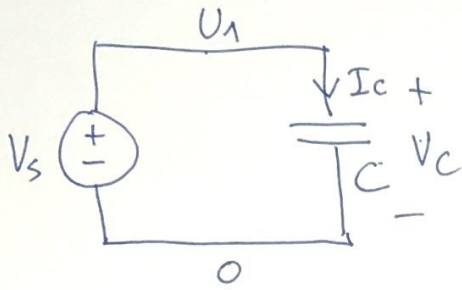
$$I_s \cdot t = C \cdot (U_1(t) - U_1(0))$$

$$U_1(t) = \frac{I_s}{C} \cdot t + U_1(0)$$



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Simple circuit #2:

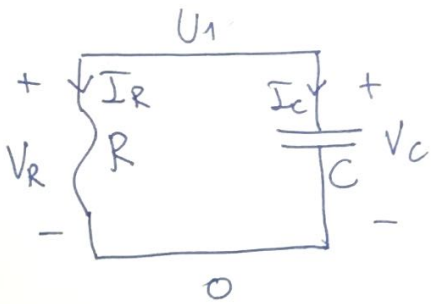


$$V_s = V_c \begin{cases} U_1 = V_s & (\text{voltage elem def}) \\ U_1 = V_c & (\text{voltage def}) \end{cases}$$

$$I_c = C \frac{dV_c}{dt} \quad (\text{capacitance elem. def})$$

$$I_c = C \frac{dV_c}{dt} = C \cdot \frac{dV_s}{dt} = 0$$

Simple circuit #3:



steady-state means
when voltages settle

Looking for U_1 value when
 $V_c = \text{const}$ (steady-state)

$$\Downarrow \\ I_c = C \frac{dV_c}{dt} = 0$$

$$\text{KCL: } I_c + I_R = 0$$

$$\Downarrow \\ I_R = 0$$

$$\text{Ohm's law: } V_R = I_R R = 0$$

$$\text{voltage def: } U_1 - 0 = V_R = 0$$

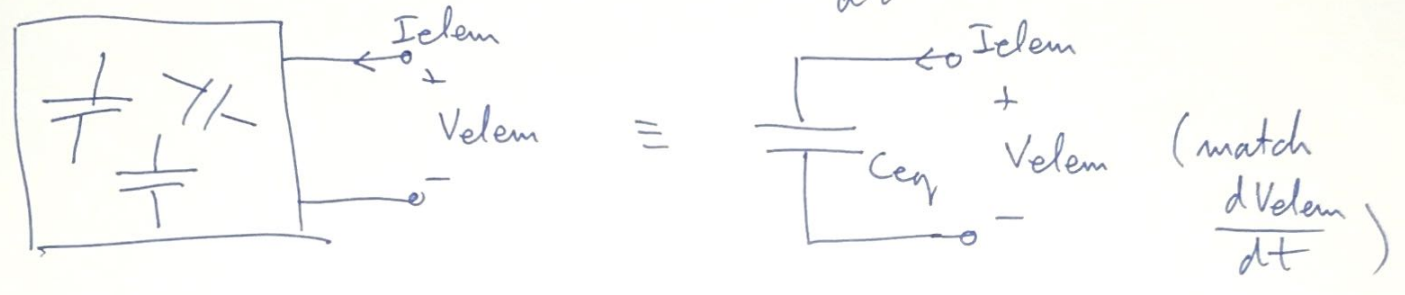
$$\Downarrow \\ \boxed{U_1 = 0}$$

Equivalent circuits with capacitors:

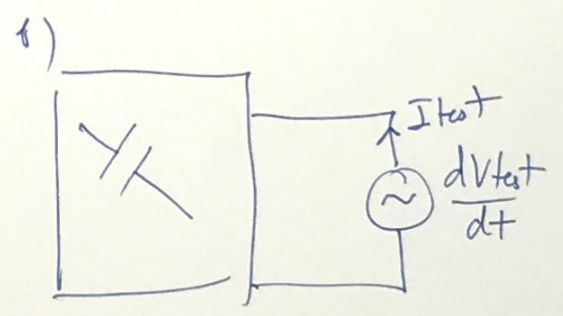
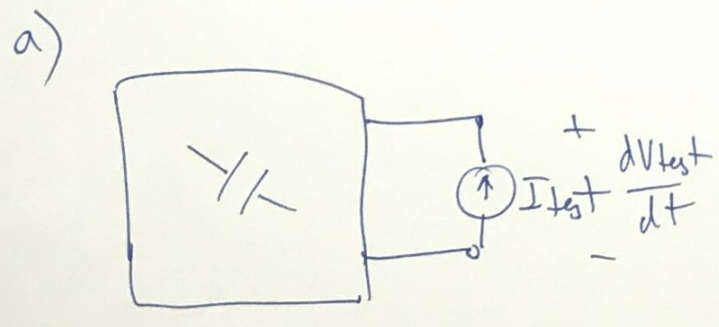
* Capacitor - only circuits

* step 1: find V_{th} or I_{N0} No source

* step 2: $C_{eq} = \frac{I_{elem}}{\frac{dV_{elem}}{dt}}$

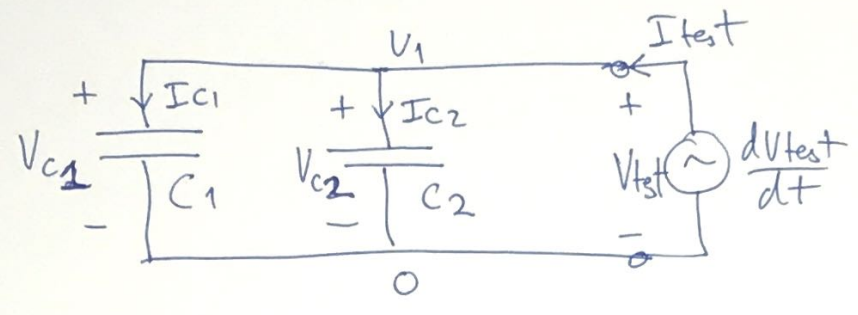


- a) Apply I_{test} and measure $\frac{dV_{test}}{dt}$
 - or
 - b) Apply $\frac{dV_{test}}{dt}$ and measure I_{test}
- $C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$



(25)

Example 1:



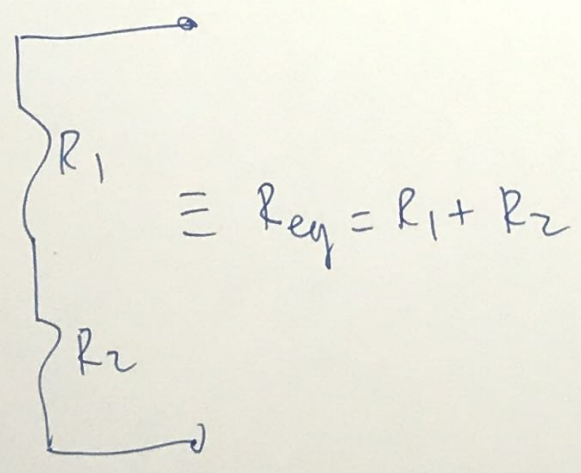
$V_{c1} = U_1$, $V_{c2} = U_1$ and $U_1 = V_{test} \Rightarrow \frac{dU_1}{dt} = \frac{dV_{test}}{dt}$

elem. df: $I_{c1} = C_1 \frac{dV_{c1}}{dt} = C_1 \frac{dU_1}{dt} = C_1 \frac{dV_{test}}{dt}$
 elem. df: $I_{c2} = C_2 \frac{dV_{c2}}{dt} = C_2 \frac{dU_1}{dt} = C_2 \frac{dV_{test}}{dt}$

KCL: $I_{test} = I_{c1} + I_{c2} = C_1 \frac{dV_{test}}{dt} + C_2 \frac{dV_{test}}{dt}$

$I_{test} = (C_1 + C_2) \frac{dV_{test}}{dt}$

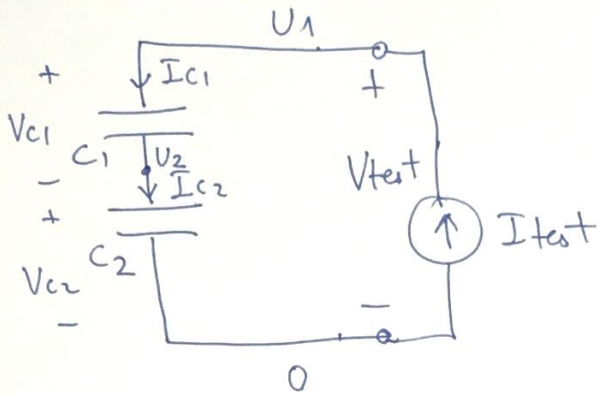
$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = C_1 + C_2$



Q6

Example 2:

"Capacitors in series"



KCL:

$$I_{c1} = I_{c2} = \underline{I_{test}}$$

elements:

$$I_{c2} = C_2 \frac{dV_{c2}}{dt}$$

$$I_{c1} = C_1 \frac{dV_{c1}}{dt}$$

voltage def:

$$V_{c2} = U_2 - 0 = U_2$$

$$V_{c1} = U_1 - U_2$$

$$V_{test} = U_1 - 0$$

$$I_{test} = C_2 \frac{dU_2}{dt}$$

$$\frac{dV_{c1}}{dt} = \frac{dU_1 - dU_2}{dt} = \frac{I_{test}}{C_1}$$

$$\frac{dU_1}{dt} = \frac{dU_2}{dt} + \frac{I_{test}}{C_1} = \frac{I_{test}}{C_2} + \frac{I_{test}}{C_1}$$

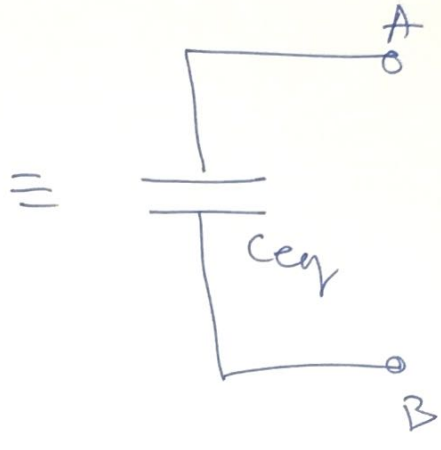
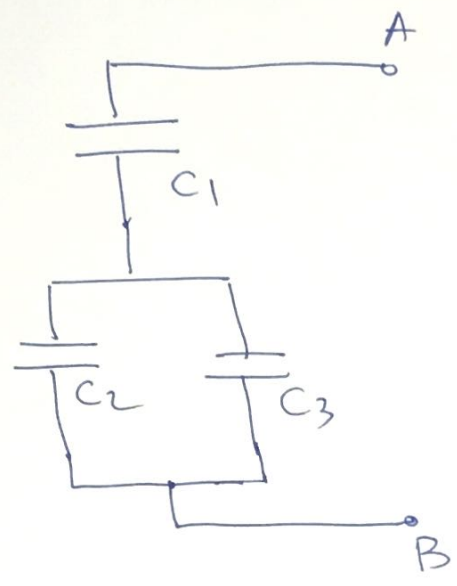
$$\frac{dU_1}{dt} = \frac{dV_{test}}{dt} = I_{test} \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

$$C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2} = C_1 \parallel C_2$$

$$C_{eq} = C_1 \parallel C_2 \quad (\parallel - \text{parallel mathematical operator})$$

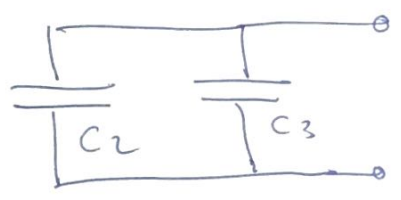
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Example # 3 :

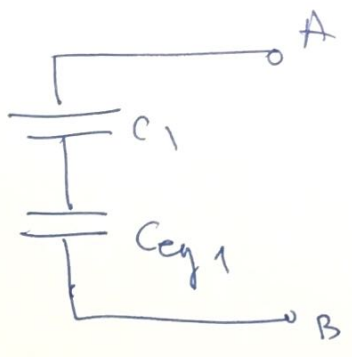


$$C_{eq} = C_1 \parallel (C_2 + C_3)$$

$$= (C_2 + C_3) \parallel C_1$$

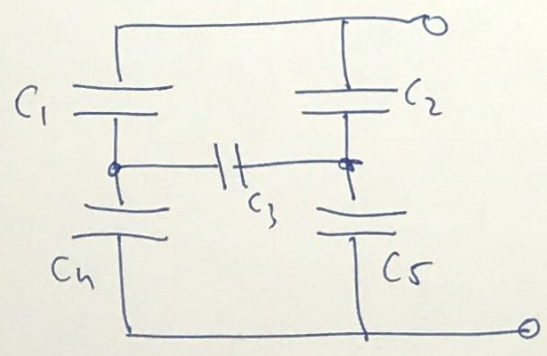


$$\Rightarrow C_{eq1} = C_2 + C_3$$



$$C_{eq} = C_1 \parallel C_{eq1}$$

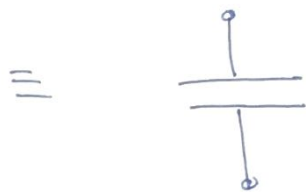
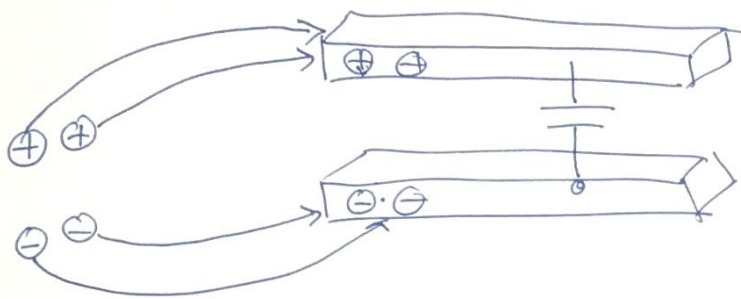
challenge det :



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16A Capacitor physics

Capacitor: Any two conductors separated by an insulator (cannot carry current)



$$\underline{Q = C \cdot V}$$

$$\frac{dE}{dq} = V \Rightarrow E = \frac{1}{2} CV^2$$

Capacitor is a "bucket" for charge

Capacitor depends on: 1) geometry of the conductors
2) material properties of the insulator

side view:

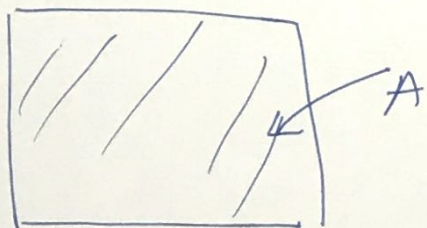


- ϵ - permittivity $\left[\frac{F}{m}\right]$

$$\epsilon_0 = 8.85 \frac{\mu F}{m}$$

$$\mu = \mu_0 \epsilon_0 = 10^{-12}$$

top view:



$$C = \epsilon \frac{A}{d}$$

\uparrow \uparrow \uparrow
 $[F]$ $\left[\frac{F}{m}\right]$ $\left[\frac{m^2}{m}\right]$