EECS 16A Designing Information Devices and Systems I
Fall 2019 Discussion 1B

1. Systems of Equations

Solve the following systems of equations, or if there is no solution, explain why. Can you visualize these geometrically?

(a)

$$\begin{cases} 2x+y=6 \\ 3x-2y=2 \end{cases}$$

Answer:

There are many ways to solve systems of linear equations, here we will use substitution.

$$\begin{aligned} 2x+y=6 &\implies y=6-2x \\ 3x-2(6-2x)&=2 \\ 7x&=14 \\ x&=2 \\ y&=6-2(2)=2 \end{aligned}$$

(b)

$$\begin{cases} x+y+z=2 \\ x-y=1 \\ 2y+z=1 \end{cases}$$

Answer:

To solve this system of linear equations, we will begin by subtracting the second equation from the first equation.

$$2y+z=1$$

Notice that this equation is the same as equation 3. Therefore, the system of linear equations does not have a unique solution, infact it has infinitely many solutions.

The set of solutions can be described by a set of parameteric equations. To find the equations, we begin by chosing a parameter t, and set one of the variables equal to t, we chose z. Then we can write the other variables in terms of z and thus t.

$$
\begin{aligned}
z &= t \\
y &= \frac{1-z}{2} = \frac{1-t}{2} \\
x &= 2 - y - z = 2 - \frac{1-t}{2} - t = \frac{3}{2} - \frac{1}{2}t
\end{aligned}
$$

(c)

$$
\begin{cases} 6x + 2y = 15 \\ 3x + y = 7 \end{cases}
$$

Answer:

Notice that if you multiply the second equation by 2, you obtain $6x+2y = 14$. This is inconsistent with the first equation, as $6x+2y = 15$, therefore there is no solution.

2. Vectors Introduction to vectors and vector addition.

Definitions:
Vector: An ordered list of elements - for example:

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$$

$\mathbb{R}$ or $\mathbb{R}^1$: The set of all real numbers (i.e. the real line)
$\mathbb{R}^2$: The set of all two-element vectors with real numbered entries (i.e. plane of 2×1 vectors) - for example:

$$\vec{v} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \in \mathbb{R}^2$$

$\mathbb{R}^3$: The set of all three-element vectors with real numbered entries (i.e. 3-space of 3×1 vectors) - for example:

$$\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^3$$

$\mathbb{R}^n$: The set of all n-element vectors with real numbered entries (i.e. n-space of $n \times 1$ vectors)

(a) Are the following vectors in $\mathbb{R}^2$?

i. $\begin{bmatrix} 3 \\ 6 \end{bmatrix}$

ii. $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$

Answer:

i. Yes, it is a two element vector of real numbered entries.
ii. Yes, it is a two element vector of real numbered entries.

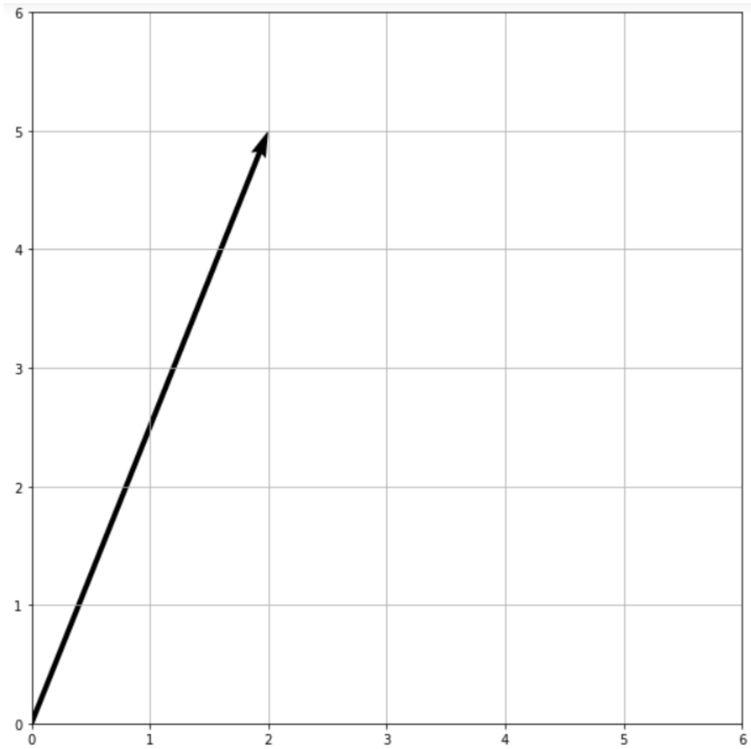
(b) Graphically show the vectors:

i. $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

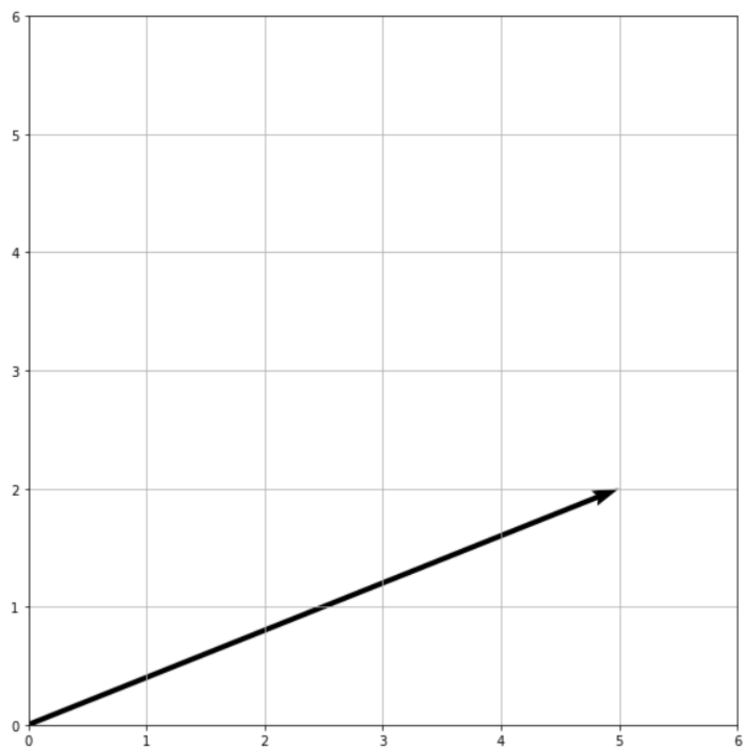
ii. $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$

Answer:

i.

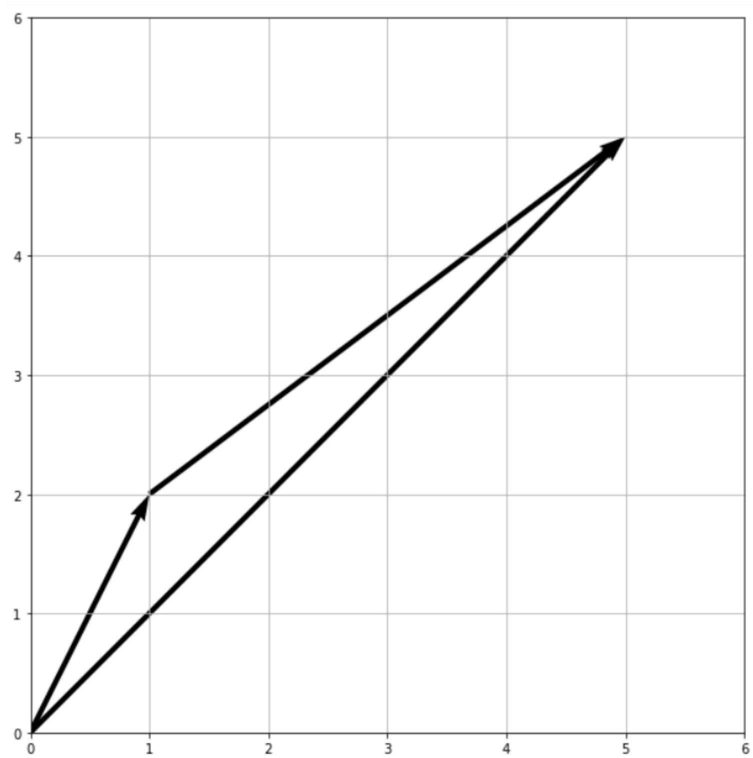


ii.



(c) Graphically show the vector sum and check your answer algebraically:

$$\begin{bmatrix}1\\2\end{bmatrix}+\begin{bmatrix}4\\3\end{bmatrix}$$



Answer:

$$\begin{bmatrix}1\\2\end{bmatrix}+\begin{bmatrix}4\\3\end{bmatrix}=\begin{bmatrix}5\\5\end{bmatrix}$$