EECS 16A Designing Information Devices and Systems I
Fall 2019 Discussion 2A

Definition: A **linear combination** (also sometime referred to as a weighted combination) of vectors, $\vec{v}_1, \vec{v}_2 \ldots, \vec{v}_n$, is given as a weighted summation of the vectors,

$$\alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2 + \cdots + \alpha_n \vec{v}_n, \tag{1}$$

where weights, $\alpha_1, \alpha_2, \ldots, \alpha_n \in \mathbb{R}$, scale the vectors.

1. Finding The Bright Cave

Nara the one-handed druid and Kody the one-handed ranger find themselves in dire straits. Before them is a cliff with four cave entrances arranged in a square: two upper caves and two lower caves. Each entrance emits a certain amount of light, and the two wish to find exactly the amount of light coming from each cave. Here's the catch: after contracting a particularly potent strain of ghoul fever, our intrepid heroes are only able to see the total intensity of light before them (so their eyes operate like a single-pixel camera). Kody and Nara are capable adventurers, but they don't know any linear algebra – and they need your help.

Kody proposes an imaging strategy where he uses his hand to completely block the light from two caves at a time. He is able to take measurements using the following four masks (black means the light is blocked from that cave):

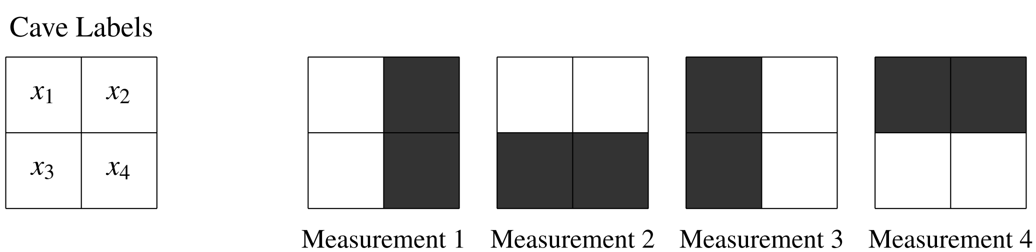


Figure 1: Four image masks.

(a) Let $\vec{x}$ be the four-element vector that represents the magnitude of light emanating from the four cave entrances. Write a matrix $\mathbf{K}$ that performs the masking process in Figure 1 on the vector $\vec{x}$, such that $\mathbf{K}\vec{x}$ is the result of the four measurements.

(b) Does Kody's set of masks give us a unique solution for all four caves' light intensities? Why or why not?

(c) Nara, in her infinite wisdom, places her one hand diagonally across the entrances, covering two of the cave entrances. However, her hand is not wide enough, letting in 50% of the light from the caves covered and 100% of the light from the caves not covered. The following diagram shows the percentage of light let through from each cave:

50%	100%
100%	50%

Does this additional measurement give them enough information to solve the problem? Why or why not?

2. How many solutions?

(a) We are given a system of equations as the augmented matrix:

$$\left[\begin{array}{ccc|c} 2 & 6 & 4 & 10 \\ 1 & -3 & 3 & 13 \\ 0 & 0 & 3 & 12 \end{array}\right] \tag{2}$$

Use Gaussian elimination to determine how many solutions the system of equations has.

i. Unique solution
ii. Infinite solutions
iii. No solutions

(b) We are given a system of equations as the augmented matrix: $\left[\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 0 & 0 & 2 & 1 \end{array}\right]$. Use Gaussian elimination to determine how many solutions the system of equations has.

i. Unique solution
ii. Infinite solutions
iii. No solutions

(c) We are given the system of equations:

$$\begin{bmatrix} 2x & + & 4y & + & 2z & = & 8 \\ x & + & y & + & z & = & 6 \\ x & - & y & - & z & = & 4 \end{bmatrix} \tag{3}$$

Use Gaussian elimination to determine how many solutions the system of equations has.

i. Unique solution
ii. Infinite solutions
iii. No solutions

(d) We are given the system of equations:

$$\begin{bmatrix} x & + & y & + & 2z & = & 2 \\ & & y & + & z & = & 0 \\ 2x & + & y & + & 3z & = & 4 \end{bmatrix} \tag{4}$$

Use Gaussian elimination to determine how many solutions the system of equations has.

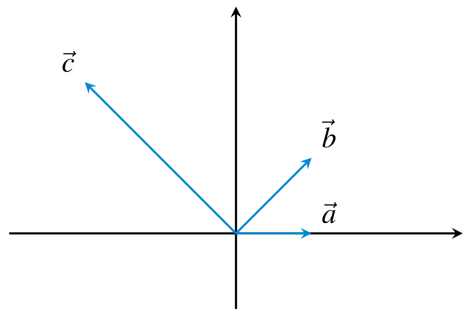
i. Unique solution
ii. Infinite solutions

iii. No solutions

(e) True or False: A system of equations with more equations than unknowns will always have either infinite solutions or no solutions.

3. Visualizing Span

We are given a point $\vec{c}$ that we want to get to, but we can only move in two directions: $\vec{a}$ and $\vec{b}$. We know that to get to $\vec{c}$, we can travel along $\vec{a}$ for some amount α, then change direction, and travel along $\vec{b}$ for some amount β. We want to find these two scalars α and β, such that we reach point $\vec{c}$. That is, $\alpha\vec{a}+\beta\vec{b}=\vec{c}$.



(a) First, consider the case where $\vec{a}=\begin{bmatrix}1\\0\end{bmatrix}$, $\vec{b}=\begin{bmatrix}1\\1\end{bmatrix}$, and $\vec{c}=\begin{bmatrix}-2\\2\end{bmatrix}$. Draw these vectors on a sheet of paper. Now find the two scalars α and β, such that we reach point $\vec{c}$. What are these scalars if we use $\vec{a}=\begin{bmatrix}1\\1\end{bmatrix}$ and $\vec{b}=\begin{bmatrix}2\\1\end{bmatrix}$ instead?

(b) Formulate the system of equations as a matrix to find the unknowns, α,β, in terms of the vectors $\vec{a},\vec{b},\vec{c}$.