Definition: If $A\vec{x} = \lambda \vec{x}$, then $\lambda \in \mathbb{R}$ is called an eigenvalue of $A$. $\vec{x}$ belongs to the eigenspace of $A$ corresponding to eigenvalue $\lambda$. All vectors $\vec{x}$ in the eigenspace are called eigenvectors corresponding to the eigenvalue $\lambda$.

1. Mechanical Eigenvalues and Eigenvectors

In each part, find the eigenvalues of the matrix $M$ and the associated eigenvectors. If the inverse of $M$ exists, find it.

(a) $M = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(b) $M = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

(c) (PRACTICE) $M = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

(d) (PRACTICE) $M = \begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$

2. Steady State Reservoir Levels

We have 3 reservoirs: $A$, $B$ and $C$. The pumps system between the reservoirs is depicted in Figure 1.

![Reservoir pumps system](image)

(a) Write out the transition matrix $T$ representing the pumps system.

(b) You are told that $\lambda_1 = 1$, $\lambda_2 = \frac{-\sqrt{2} - 1}{10}$, $\lambda_3 = \frac{\sqrt{2} - 1}{10}$ are the eigenvalues of $T$. Find a steady state vector $\vec{x}$, i.e. a vector such that $T\vec{x} = \vec{x}$. 

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3. Proofs

(a) Let $A$ be an invertible matrix. Show that if $\lambda$ is an eigenvalue of $A$, then $\frac{1}{\lambda}$ is an eigenvalue of $A^{-1}$. 