1. Power Analysis

(a) Find the power dissipated by each element in the circuit above. Remember to label voltages using passive sign convention.

(b) Use $R = 5k\Omega$, $V_s = 5V$, and $I = 5mA$. Calculate $P_V$, $P_I$, and $P_R$.

(c) Repeat part (b) but change the value $I$ of the current source such that it dissipates $40mW$. Calculate $I$, $P_V$, $P_I$, and $P_R$.

2. Maximum Power Transfer

Smartphones use "bars" to indicate strength of the cellular signal. Few "bars" translate to slow or no connectivity.

But what do these "bars" actually stand for? Voltage, current? Well, not quite. Good radio - and a cellular modem is nothing but a particular kind of radio - reception depends on the power received from the transmitter.

In this assignment we design a receiver that maximizes the power received, and hence connection speed.

The figure below shows electronic circuit models for the antenna (a) and the receiver (b). The antenna consists of source $V_s$, typically in the range of micro- or milli-Volts ($10^{-6}$ and $10^{-3}$, respectively) depending on transmitter strength and resistance $R_s$, usually $50\Omega$ or $75\Omega$, depending on the particular antenna design.

The radio receiver is represented by resistor $R_L$ and chosen carefully by the designer to maximize the received power (i.e. the "number of bars").
Figure 1: Electronic circuit models for radio antenna and receiver.

It is important to understand that these are models. For example, the complete receiver circuit consists of many more elements. Likewise, the antenna consists of several appropriately shaped conductors. The resistor $R_s$ is nowhere to be found, and neither is $V_s$ physically present (i.e. you cannot connect a wire to it). However, these simple models act like the devices they represent. In other words, the voltages and currents at their terminal are identical to the voltages and currents at the terminals of the actual circuits.

Models are very important in engineering design for their ability to abstract away details when they are not needed and are the key to successful design of complex systems.

We will discuss the use and properties of electronic circuit models further in class.

Use the following component values for your calculations: $V_s = 100 \mu V$, and $R_s = 50 \Omega$.

(a) Find the value of $R_L$ that maximizes the voltage $V_L$ across resistor $R_L$. Calculate the values of $V_L$, $I_L$, and the power $P_L$ delivered to (i.e. dissipated in) resistor $R_L$.

(b) Find the value of $R_L$ that maximizes the current $I_L$ through resistor $R_L$. Calculate the values of $V_L$, $I_L$, and the power $P_L$ delivered to resistor $R_L$.

(c) Find the value of $R_L$ that maximizes the power $P_L$ delivered to resistor $R_L$. Calculate the values of $V_L$, $I_L$, and the power $P_L$ delivered to resistor $R_L$. (Hint: The power optimization is best performed algebraically by setting the derivative of $P_L$ with respect to $R_L$ to zero. Alternatively you can do the optimization graphically. Plot $P_L$ versus $R_L$ and find the maximum.)

(d) What’s the best value of $R_L$ that optimizes cellular connectivity (i.e. provides the most amount of received power)?

The next step is to design the receiver circuit such that it behaves like a resistor $R_L$ and extracts the information sent. How to do this is taught in EE142, "Integrated Circuits for Communications."

3. Cell Phone Battery

As great as smartphones are, one of their drawbacks is that their batteries don’t last a long time. For example, a Google Pixel phone, under typical usage conditions (internet, a few cat videos, etc.) uses 0.3W. We will model the battery as an ideal voltage source (which maintains a constant voltage across its terminals regardless of current) except that we assume that the voltage drops abruptly to zero when the battery is discharged (in reality the voltage drops gradually, but let’s keep things simple).
Battery capacity is specified in mAh, which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. The Pixel’s battery has a battery capacity of 2770mAh at 3.8V. For example, this battery could provide 1000mA (or 3.8W) for 2.77 hours before the voltage drops from 3.8V to zero.

(a) How long will a Pixel’s full battery last under typical usage conditions?
(b) How many coulombs of charge does the battery contain? How many usable electrons worth of charge are contained in the battery when it is fully charged? (An electron has $1.602 \times 10^{-19}$ C of charge.)
(c) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a Ws.
(d) Suppose PG&E charges $0.12 per kWh. Every day, you completely discharge the battery (meaning more than typical usage) and you recharge it every night. How much will recharging cost you for the month of October (31 days)?
(e) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). We will model the battery and its internal circuitry as a resistor $R_{bat}$. We now wish to charge the battery by plugging into a wall plug. The wall plug can be modeled as a 5V voltage source and 200mΩ resistor, as pictured in Figure 2. What is the power dissipated across $R_{bat}$ for $R_{bat} =$ 1 mΩ, 1 Ω, and 10kΩ? (i.e. how much power is being supplied to the phone battery as it is charging?). How long will the battery take to charge for each of those values of $R_{bat}$?

![Figure 2: Model of wall plug, wire, and battery.](image)

4. Measuring Voltage and Current

In order to measure quantities such as voltage and current, engineers use voltmeters and ammeters. A simple model of a voltmeter is a resistor with a very high resistance, $R_{VM}$. The voltmeter measures the voltage across the resistance $R_{VM}$. The measured voltage is then relayed to a microprocessor (such as the MSP430s used in Lab).

This model of an voltmeter is shown in Figure 3. Let us explore what happens when we connect this voltmeter to various circuits to measure voltages. Throughout this problem assume $R_{VM} = 1M\Omega$. Recall that the SI prefix $M$ or Mega is $10^6$.

![Figure 3: Our model of a voltmeter, $R_{VM} = 1M\Omega$](image)
(a) Suppose we wanted to measure the voltage across $R_2$ ($v_{out}$) produced by the voltage divider circuit shown in Figure 4 on the left. The circuit on the right in Figure 4 shows how we would connect the voltmeter across $R_2$. Assume $R_1 = 100\,\Omega$ and $R_2 = 200\,\Omega$.

First calculate the value of $v_{out}$. Then calculate the voltage the voltmeter would measure, i.e. $v_{meas}$.

![Figure 4: Left: Circuit without the voltmeter connected, Right: Voltmeter measuring voltage across $R_2$](image)

(b) Repeat part (a), but now $R_1 = 10\,M\Omega$ and $R_2 = 10\,M\Omega$. Is this voltmeter still a good tool to measure the output voltage?

(c) Now suppose we are working with the same circuit as in Part (a), but we know that $R_2 = R_1$. What is the maximum value of $R_1$ that ensures that the difference between voltage measurement of the voltmeter ($v_{meas}$) and the actual value ($v_{out}$) remains within $\pm10\%$ of $v_{out}$?

(d) Using the combination of our voltmeter and an additional resistor $R_x$, we can make an ammeter and measure the current through an element. Using the circuit shown in Figure 5 where $R_x = 1\,\Omega$, then the measured current through $R_x$ is $I_{meas} = \frac{V_{VM}}{R_x}$ where $V_{VM}$ is the voltage across the voltmeter.

In Figure 6 the voltmeter is connected to measure the current through resistor $R_1 = 1\,k\Omega$. For the circuit on the left, find the current through $R_1$ without the voltmeter connected (i.e. $I_1$). Then, for the circuit on the right, find the current measured by the voltmeter when it is connected as an ammeter (i.e. $I_{meas}$).

![Figure 5: The voltmeter combined with resistor $R_x$ to function as an ammeter (i.e. to measure current), $R_{VM} = 1M\Omega$.](image)
Figure 6: Circuits for Part (d) Left: Original circuit; Right: Circuit with the voltmeter connected as an ammeter.

(e) What is the minimum value of $R_1$ that ensures the difference between current measurement ($I_{\text{meas}}$) and the actual value ($I_1$) stays within $\pm 10\%$ of $I_1$?

5. Equivalent Resistance (PRACTICE)

(a) Find the equivalent resistance looking in from points $a$ and $b$.

(b) Find the equivalent resistance looking in from points $a$ and $b$.

(c) Find the equivalent resistance looking in from points $a$ and $b$. 
6. Circuits Practice (PRACTICE)

(a) What does KVL tell you about $V_1$ and $V_2$ for any elements connected to the same pair of nodes?

(b) What does KCL tell you about $I_1$ and $I_2$ for any two elements connected to a node with nothing else connected to that node?

(c) Find $R_{ab}$, the equivalent resistance between terminals $a$ and $b$. Give your answer as a number, or an expression involving no more than one use of $\parallel$.

(d) Find $I_x$. (Hint: Reference Discussion 7B)

7. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID’s. (In case of homework party, you can also just describe the group.) How did you work on this homework?