

1. Trigonometric Identities

Prove the following identities:

(a) $\cos t = \frac{1}{2}(e^{jt} + e^{-jt})$

(b) $\sin t = \frac{1}{2}(e^{jt} - e^{-jt})$

(c) $\cos(t \pm \frac{\pi}{2}) = \mp \sin t$

(d) $\sin(t \pm \frac{\pi}{2}) = \pm \cos t$

(e) $2 \sin t \cos t = \sin 2t$

$$(f) \cos^2 t = \frac{1}{2}(1 + \cos 2t)$$

$$(g) \sin^2 t = \frac{1}{2}(1 - \cos 2t)$$

$$(h) \sin t_1 \sin t_2 = \frac{1}{2}(\cos(t_1 - t_2) - \cos(t_1 + t_2))$$

$$(i) \cos t_1 \cos t_2 = \frac{1}{2}(\cos(t_1 - t_2) + \cos(t_1 + t_2))$$

2. Compute the following:

(a)

$$\int_{-T}^T \cos \frac{n\pi t}{T} \cos \frac{m\pi t}{T} dt$$

(b)

$$\int_{-T}^T \sin \frac{n\pi t}{T} \sin \frac{m\pi t}{T} dt$$

(c)

$$\int_{-T}^T \cos(\omega_1 t) \sin(\omega_2 t) dt$$