EECS 16A Designing Information Devices and Systems I
Spring 2015

Discussion 3F

1. Intersection of Subspaces

Given a vector space $(\mathbb{V}, \mathbb{F})$, a non-empty subset $\mathbb{W}$ of $\mathbb{V}$ that is closed under addition and multiplication, such that $(\mathbb{W}, \mathbb{F})$ is a vector space itself, is called a **subspace** of $(\mathbb{V}, \mathbb{F})$.

Prove that the intersection of two subspaces of a vector space $(\mathbb{V}, \mathbb{F})$ is a subspace of the same vector space $(\mathbb{V}, \mathbb{F})$.

2. Matrix Multiplication

(a) Let $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ 3 & -1 \end{bmatrix}$. Compute AB and BA. Are they equal?

(b) Let A and B be the 2×2 rotation matrices for angles θ and ϕ, respectively. Compute AB and BA. Are they equal?

Complex Numbers

Motivation for complex numbers and more importantly $i = \sqrt{-1}$ came about when there was an interest to find roots for polynomials of the form $x^2 + c = 0$, where $c \in \mathbb{R}$ and $c > 0$. By extending the real numbers field $\mathbb{R}$ to the complex numbers field $\mathbb{C}$, we can represent roots of every polynomial equation.

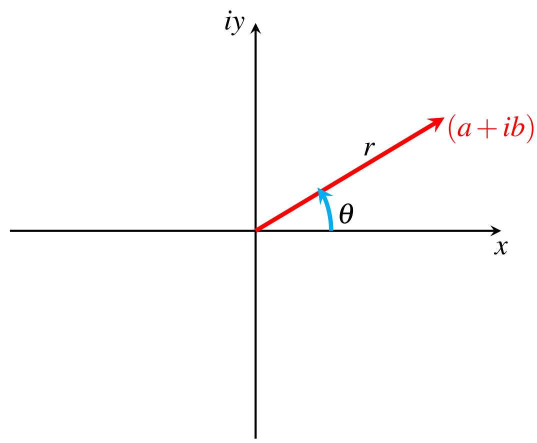
A complex number c is an ordered pair of real numbers (a, b) that is represented as

$$c = a + ib$$

where a is the "real" part of c and b is the "imaginary" part of c.

Coordinate Axis Representation of Complex Numbers

A complex number $c = a + ib$ can be plotted on the X-Y plane as follows:



The modulus (or "length") of a complex number $c = a + ib$ is equal to $\sqrt{a^2 + b^2}$. Therefore, a complex number can also be represented in the polar form as $c = r(\cos\theta + i\sin\theta)$ where $r = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$. This is alternatively and very popularly represented as $c = re^{i\theta}$, where $e^{i\theta} = \cos\theta + i\sin\theta$.

Properties of Complex Numbers

- Equality of complex numbers: Complex numbers $c = a + ib$ and $c' = a' + ib'$ are equal if and only if $a = a'$ and $b = b'$. In polar representation, this means $r = r'$ and $\theta = \theta' + 2k\pi$ where k is an integer.
- Sum of complex numbers: if $c = a + ib$ and $c' = a' + ib'$, then $c + c' = (a + a') + i(b + b')$.
- Product of complex numbers: if $c = a + ib$ and $c' = a' + ib'$, then $c \times c' = (a \cdot a' - b \cdot b') + i(a \cdot b' + a' \cdot b)$. In polar representation $c \times c' = (r \cdot r')e^{i(\theta + \theta')}$.
- Conjugate of a complex number: if $c = a + ib$, then the complex conjugate of the number c is given by $\text{conj}(c) = c^* = \overline{a + ib} = a - ib$. In polar representation $\text{conj}(c) = c^* = re^{-i\theta}$.
- Inverse of a complex number: if $c = a + ib$, then the inverse is the complex number which when multiplied with c gives 1. Thus, $\text{inv}(c) = \frac{1}{c} = \frac{a - ib}{a^2 + b^2}$. In polar representation $\text{inv}(c) = \frac{1}{c} = r^{-1}e^{-i\theta}$.

1. **Natural Power of a Complex Number**

   If c is a complex number given by $c = re^{i\theta}$, then prove that $c^n = r^n e^{in\theta}$.

2. **Roots of a Polynomial**

   (a) Find the roots of $f(x) = x^2 + 4x + 5$

   (b) Find the roots of $f(x) = x^2 + 4x + 4$

   Is there a relationship between the roots of a function and its coefficients?