EECS 16A Designing Information Devices and Systems I
Spring 2015

Midterm I Sample Problems

1. **Stealthy Baking**

   There is a new bakery opening in Berkeley, Engineering Edibles, that is all the rage. They make three types of cookies Channing Chocolate, Bountiful Bancroft and Dramatic Durant. However, they are very secretive about their recipes and want to make sure they don't leak out, but a group of Berkeley students decides to try and hack their system. The three key ingredients used are eggs, butter and sugar, the question to understand is what is the proportion in each type of cookie. On a given day the bakery produces 60 Channing Chocolate, 40 Bountiful Bancroft and 50 Dramatic Durant cookies. CC cookies cost \$2 each, BB cookies cost \$3 and DD cookies cost \$4. \$1 of the cost comes from labor and other ingredients, but all the rest of is just from the eggs, butter and sugar. One egg costs 10 cents, 100 grams of sugar costs 50 cents and 100 grams of butter costs a dollar.

   A master-taster student can immediately tell the amount of sugar in a cookie by biting into it. And from this he realizes that there must be 1 gram of sugar in the CC cookies and BB cookies but 2 grams of sugar in each of the DD cookies. Based on the texture of the cookie, he is also able to figure out the butter in the CC and BB cookies is exactly the same, but cannot determine the amount.

   Finally, the team goes dumpster diving and finds that there are 135 eggs used on a day, 200 grams of sugar used and 300 grams of butter used. How can they now found how much sugar, butter and eggs are used in each cookie? Set this up as a linear system of equations. Solve them, or give a detailed procedure which you would use to solve the system.

2. **Projections**

   Consider the following projection system. We place our camera at the origin, and our object a triangle, in the plane that is normal to the x-axis and passes through the point $(1,0,0)$. The plane of the screen passes through $(5,0,0)$ and is also normal to the x-axis. The projection of the three vertices of the triangle are $(5,0,0),(5,3,0)$ and $(5,0,4)$.

   (a) Express the relationship between the vertices of the object and it's projection as a matrix operation. Find the position of the vertices of the original triangle.

   (b) Now there is a new object, a square that is placed in the original plane. It's vertices are $(1,0,0)$, $(1,1,0),(1,1,1),(1,0,1)$. Find the vertices of the projection of this square.

3. **Eigenvalues and eigenvectors**

   We've talked in class about eigenvectors of a matrix A, and how special they are: pre-multiply an eigenvector by the matrix A, the result is this eigenvector, scaled by the corresponding eigenvalue. Now suppose A is a 2×2 matrix, and pre-multiplication of *any* point in $\mathbb{R}^2$ by A corresponds to reflecting the point across the line defined by the vector $[2,1]^T$.

   (a) By thinking only about the effect that A has on points in $\mathbb{R}^2$, determine the eigenvectors of A. What are the eigenvalues corresponding to these eigenvectors?

   (b) From its eigenvalues and eigenvectors, determine A.