EECS 16A Designing Information Devices and Systems I
Spring 2015 # Homework 10

This homework is due April 30, 2015 at 5PM.

1. **The Phantom of the Opera**

   You just watched a show of "The Phantom of the Opera" and really liked it! In particular, you loved the title track and wanted to explore a piece of it. You want to check out the effects that different filters have on the song and the resulting spectrum. Download the files `phantom_mono.mat` and `prob10.ipynb`.

   (a) Listen to the music and plot the spectrum of the music piece.

   (b) Compute the frequency response of the system defined by the impulse response $h_1[n] = \frac{1}{2}(\delta[n] + \delta[n-2])$ (i.e., compute the response of this system to the input $e^{j\omega n}$ to get the system response $H(e^{j\omega})$) and sketch the magnitude of the frequency response of this filter.

   (c) Pass the music through the filter $h_1[n] = \frac{1}{2}(\delta[n] + \delta[n-2])$ and listen to the filtered music. What happens to the spectrum of the resultant signal? What kind of a filter do you think this is? (Hint: the resultant signal will be $y[n] = (x[n] + x[n-2])/2$. Just shift the signal using `np.roll` and add it to the signal. Do not worry about edge effects)

   (d) Sketch the magnitude of the frequency response of the filter $h_2[n] = \frac{1}{2}(\delta[n] - \delta[n-2])$.

   (e) Pass the music through the filter $h_2[n] = \frac{1}{2}(\delta[n] - \delta[n-2])$ and listen to the filtered music. What happens to the spectrum of the resultant signal? What kind of a filter do you think this is? How is this filter different from the one in part (b)/(c) i.e., $h_1[n]$?

   (f) Pass the music first through the filter $h_1[n] = \frac{1}{2}(\delta[n] - \delta[n-2])$ and the resulting music through the filter $h_2[n] = \frac{1}{2}(\delta[n] + \delta[n-2])$ and then listen to it. What happens to the spectrum of the resultant signal? What kind of a filter do you think this is? Do the filtering in the reverse order. Does it sound any different?

   (g) Pass the music through the filter $h_3[n] = \frac{1}{2}(\delta[n] + \delta[n-100])$ and listen to the filtered music. What happens to the spectrum of the resultant signal? What kind of a filter do you think this is?

   (h) Plot the frequency responses of system defined by $h_3[n] = (\delta[n] + \delta[n-100])/2$ using iPython.

2. **Compression revisited!**

   In Homework 9 you explored what happens to images and sounds when the "highest" frequencies are dropped. In this problem, instead of dropping the highest frequencies, you will drop the frequency components with the smallest frequency response amplitude (Fourier coefficients).

   (a) Download the file `pharrell.mat`. Plot the spectrum of this song and then drop 80% of the frequencies with the smallest frequency response amplitude (absolute value) and reconstruct. How does the song sound compared to the compression you did in Homework 9?

   (b) Download the file `campanile.npy`. Similar to Homework 9, we can treat each column of the image as a one-dimensional array (just like audio) and repeat the same exercise as above. That is, convert each column to the frequency domain, then drop 80% of the frequencies with the smallest frequency response amplitude, and reconstruct. What do you notice about the final image once you throw most of the high frequencies away?

(c) Repeat the same exercise, except now treating each *row* of the image as a one-dimensional array.

(d) How are the compression algorithms from Homework 9 and this one different? Which one seems better?

3. LTI or not?

As discussed in lecture, if we can model a system we are interested in designing as LTI, it will allow us to use a set of very simple and powerful techniques to predict the behavior of that system with any type of input that we apply to it. In this problem we will therefore examine a few systems – some of which we have seen before in a different context – to check whether or not they are LTI. For each of these following problems, prove if they are LTI or not.

(a) Is the system show below LTI?

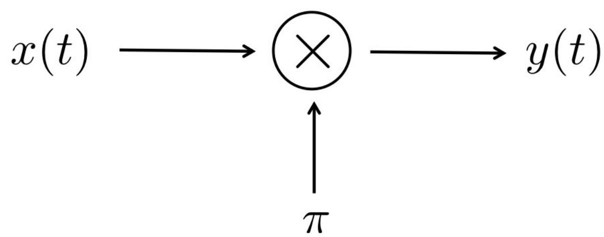


Figure 1: Figure for problem 3a

(b) Let's make a "small" modification to the system from part (a) as shown below – is this new system LTI? What operation (or system) that you've seen before are you reminded of?

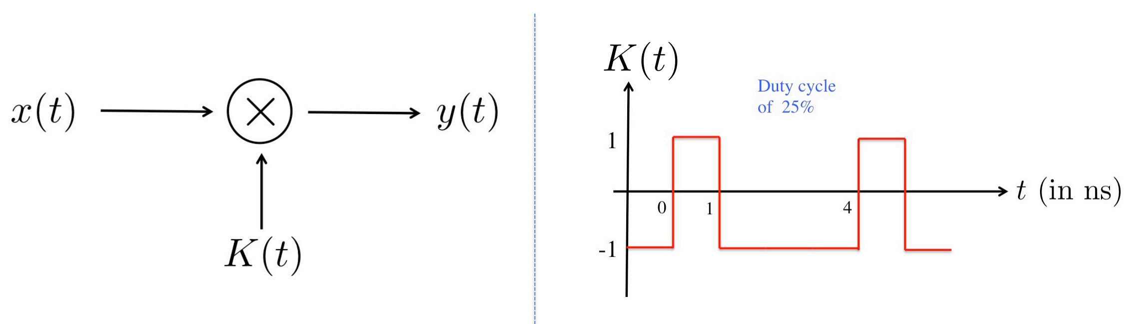
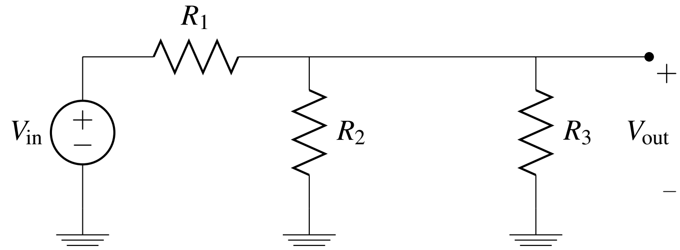


Figure 2: Figure for problem 3b

(c) (**Bonus**) Now let's make one more "small" modification the system. If we were to sample the output of the system from part (b) once every 4*ns* and starting at 0.5*ns* — i.e., $w[k] = y(k \times 4ns + 0.5ns)$ for integer $k \geq 0$ — would the system represented by $w[k] = \mathscr{H}(x(t))$ be LTI?

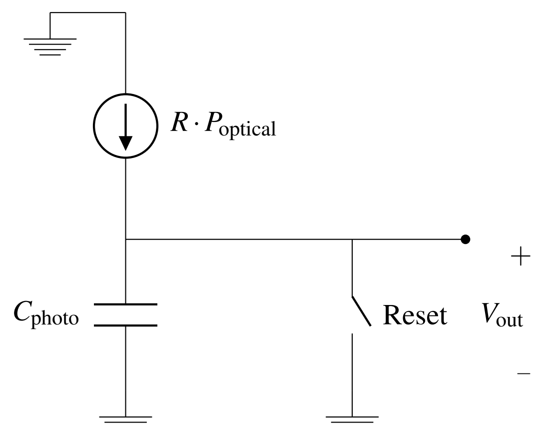
(d) For the circuit shown below, is the system represented by $V_{\text{out}} = \mathscr{H}(V_{\text{in}})$ LTI?



(e) For the same circuit shown in part (d), let P_{R_3} be the power dissipated by the resistor R_3 as a function of V_{in}. $P_{R_3} = \mathscr{H}(V_{in})$. Is this system LTI?

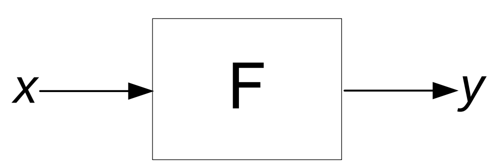
(f) For the same circuit shown in part (d), let P_{R_3} be the power dissipated by the resistor R_3 as a function of P_{in} which is the power delivered by the voltage source V_{in}. $P_{R_3} = \mathscr{H}(P_{in})$. Is this system LTI?

(g) Now let's look at a version of the photodetector circuit from the imaging module (shown below), where $P_{optical}$ is the optical intensity (power) incident on the photodetector. Assume that the optical input signal $P_{optical}$ is always 0 for $t \leq 0$ and that the reset switch is on (closed) for $t \leq 0$ and off (open) for $t \geq 0$. Is the system represented by $V_{out} = \mathscr{H}(P_{optical})$ LTI?



4. LTI system response

The impulse response of a discrete-time LTI system F



is: $h(n) = \delta(n) - \delta(n-1)$.

(a) Determine the output of the system if the input is

i. the unit-step function: $x(n) = u(n)$.

ii. a four-point discrete-time box function:

$$x(n) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3).$$

iii. a constant function: $x(n) = 1$.

iv. the sign-alternating signal $x(n) = (-1)^n$.

Be sure to provide a well-labeled sketch of all the signals involved.

(b) Based on the results of part (a), explain why the system F may aptly be called a one-dimensional *edge detector*. Does the system's response provide a clue as to which direction (up or down) each edge in the input signal is?

(c) Without determining the frequency response of the filter F, provide a reasonable conjecture as to whether you expect the filter to favor low or high frequencies. That is, do you expect the filter to be a low-pass or a high-pass filter?

(d) Provide well-labeled sketches of $|F(\omega)|$, the magnitude response of the filter. Is your magnitude response consistent with your results in part (a) and your conjecture in part (b)?

5. Sketching time!

Calculate the frequency response of these systems i.e., compute the response of this system to the input $e^{j\omega n}$ to get the system response $H(e^{j\omega})$. Sketch the magnitude of the frequency response of the following filters by hand and feel free to use iPython to help you. You should make the imporant points and the coordinate values. No need to convert the magnitude to decibels.

(a) $h[n] = \frac{1}{4}(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3])$

(b) $h[n] = \frac{1}{6}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2] + \frac{1}{6}\delta[n-3]$