# EECS 16A Designing Information Devices and Systems I
# Spring 2015 Note 5

Lecture notes by YuXuan (Andrew) Liu (02/03/2015)

## Imaging

Given a stack of bottles: Milk (M), Juice (J) or Empty (O)

$$\begin{bmatrix} M & J & O \\ M & J & O \\ M & O & J \end{bmatrix}$$

We can't "see" directly into the stack but we can shine light at different angles to get a sense of what's in each row or column.

Suppose milk absorbs 3 units of light

Juice absorbs 2 units of light

Empty bottle absorbs 2 units of light

Let's assign the amount of light absorbed at each item as $x$

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

If we shine light in a straight line, we can determine the amount of light absorbed as the sum of the light absorbed by each bottle.

The amount of light absorbed by the first row is given by:

$\sum_{i=1}^{3} x_{1i} = x_{11} + x_{12} + x_{13} = 5$

Likewise, the amount of light absorbed by the 3 columns are given the following:

$\sum x_{i1} = 6$

$\sum x_{i2} = 3$

$\sum x_{i3} = 6$

The diagonals are:

$\sum x_{ii} = 6$

$\sum x_{(4-i)i} = 3$

One such configuration that satisfies those equations is:

$$\begin{bmatrix} 3 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

From here we can deduce that the original stack looked like:

$$\begin{bmatrix} M & O & O \\ J & O & M \\ O & O & J \end{bmatrix}$$

## Reading an image

Suppose we have an image represented as a matrix:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \ldots \\ x_n \end{bmatrix}$$

One approach to read the image would be to multiply it by a matrix of 1's across the diagonal

$$\begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & \ldots & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \ldots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \ldots \\ x_n \end{bmatrix}$$

Suppose we take two measurements at the same time, i.e. $x_1 + x_2$, $x_2 + x_3$, etc.

To do this, we would use a $(n-1) \times n$ matrix of pairs of 1's across the diagonal. This will result in a $(n-1)$ vector.

$$\begin{bmatrix} 1 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 1 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & 0 & \ldots & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \ldots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_2 + x_3 \\ x_3 + x_4 \\ \ldots \\ x_{n-1} + x_n \end{bmatrix}$$

However this will not give us enough information to solve the system since we have less equations than unknowns. So, we will take the measurement of the last pixel as well:

$$\begin{bmatrix} 1 & 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 1 & \ldots & 0 & 0 \\ 0 & 0 & 1 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 1 & 0 \\ 0 & 0 & 0 & \ldots & 1 & 1 \\ 0 & 0 & 0 & \ldots & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \ldots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_2 + x_3 \\ x_3 + x_4 \\ \ldots \\ x_{n-1} + x_n \\ x_n \end{bmatrix}$$

Why might we want to use this? Because the photo detector has a threshold, and there may not be enough light to active the detector with only one pixel.

## Linear Equations

Suppose we have:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

This represents:

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$

$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$

When can we solve this system? Only when the coefficient matrix is invertible.

## Image processing

To blur an image:

$$\begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}$$

This will produce a gray image. If we want to produce finer results, we can change our interpolation weights.

$$\begin{bmatrix} 0.7 & 0.3 & 0 & 0 \\ 0 & 0.7 & 0.3 & 0 \\ 0 & 0 & 0.7 & 0.3 \\ 0 & 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.3 \\ 0.7 \\ 0.3 \\ 0.7 \end{bmatrix}$$

This will produce an image that interpolates from black to white.

## Vector spaces

A vector space $(\mathbb{V}, \mathbb{F})$ is a set of vectors $\mathbb{V}$, and a set of scalars $\mathbb{F}$ and two operators:

a. Vector addition, $+$

b. Scalar multiplication, $\cdot$

Vector addition:

a. Associative $\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$

b. Commutative $\vec{u}+\vec{v}=\vec{v}+\vec{u}$

c. Additive identity $\vec{v}+\vec{0}=\vec{v}$

d. Additive inverse $\vec{v},\vec{w}$ such that $\vec{u}+\vec{w}=\vec{0}$

Scalar multiplication:

a. Associative

b. Commutative

c. Multiplicative identity $1\cdot\vec{v}=\vec{v}$

d. Zero $0\cdot\vec{v}=\vec{0}$

And the distributive laws:

a. $a(\vec{u}+\vec{v})=a\vec{u}+a\vec{v}$

b. $(a+b)\vec{u}=a\vec{u}+b\vec{u}$