EECS 16A Designing Information Devices and Systems I
Spring 2015 Note 11

Lecture notes by Edward Wang (02/26/2015).

Resistors

Review

Ohm's law:

$V = IR$

Water pipe circuit analogy:

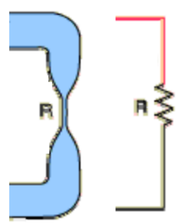


Figure 1: Water analogy for resistors.

Note that a key assumption here is **continuity** - that no charge is lost along the circuit.

Does this accurately model the real world? It turns out that this is not a big concern. Although at higher frequencies some losses are encountered, the addition of a resistor (connected to "ground") to model this loss rectifies this problem easily.

Combining resistors

What happens when we put resistors together in different ways?

For two resistors in series, the net resistance is the sum of the resistance of each of the resistors.

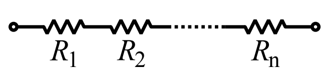


Figure 2: Resistors in series.

$$R = R_1 + R_2 + \dots$$

The assumption here is that the current stays the same while flowing through $R_1, R_2, \dots, R_n$.

For two resistors in parallel, the net resistance is the reciprocal of sum of the reciprocals of the resistance of each of the resistors.

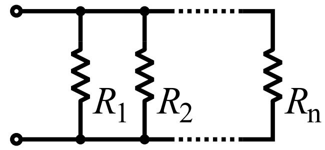


Figure 3: Resistors in parallel.

$$R = (R_1^{-1} + R_2^{-1} + \dots)^{-1}$$

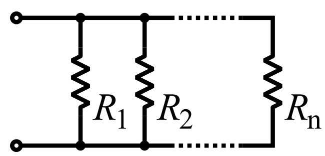
The assumption here is that the wires are perfect in each branch of the parallel circuit containing a resistor. If so, then the voltage is the same across each branch, and this fact can be used to calculate the total or equivalent resistance.

Question: What if the wire was imperfect?

Answer: Model the R_i branch with resistance $R_i + R_{iw}$, where R_{iw} is the resistance of the wire in branch R_i:

$$R = ((R_1 + R_{1w})^{-1} + (R_2 + R_{2w})^{-1} + \dots)^{-1}$$

In this class, wires will always be perfect wires with no resistance (unless we explicitly tell you otherwise)



Example: Household wiring and safety

Consider the following:

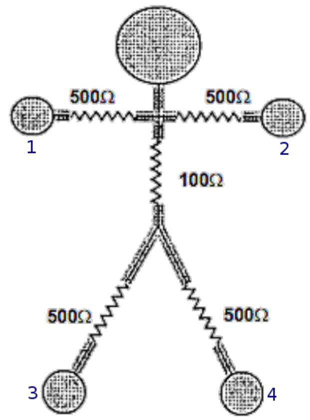


Figure 4: Simplified model of internal human body resistance.

For this exercise, ignore skin resistance, as if an electrical wire was to be inserted directly under the skin, unless otherwise specified.

Question 1: Find the resistance from sticking hand 1 into an electrical plug.

Answer 1: Where's the other end?

Question 2: Find the resistance from sticking hand 1 into an electrical plug while wearing a metal shoe touching the ground on 3 and a rubber shoe on 4.

Answer 2: $500\Omega + 100\Omega + 500\Omega = 1100\Omega$. Note that all electrical circuits are ground referenced, and that the ground at 3 is the same ground as the ground of the electrical plug.

Question 3: Find the resistance from sticking hand 1 into an electrical plug while wearing metal shoes touching the ground on both 3 and 4.

Answer 3: $500\Omega + 100\Omega + (500^{-1}\Omega + 500^{-1}\Omega)^{-1} = 850\Omega$.

Question 4: Find the resistance from sticking both hands 1 and 2 into an electrical plug while wearing metal shoes touching the ground on both 3 and 4.

Answer 4: $(500^{-1}\Omega + 500^{-1}\Omega)^{-1} + 100\Omega + (500^{-1}\Omega + 500^{-1}\Omega)^{-1} = 600\Omega$.

Question 5: Find the resistance from sticking **dry** hand 1 into an electrical plug while wearing a metal shoe touching the ground on 3 and a rubber shoe on 4. The resistance of dry skin is approximately 100000Ω.

Answer 5: $100000\Omega + 1100\Omega = 101100\Omega \approx 100000\Omega$. Notice that the resistance is approximately unchanged from that of the dry skin.

Question 6: On average, why do men have lower resistance than women?

Answer 6: Men typically have lower resistance than women mainly due to bigger arms and legs; however, note that there are other factors like percentage of body fat that can affect this as well.

Question 7: The internal resistance of the power plug is $\approx 0.5\Omega$. What is the voltage seen by the body with the same scenario as question 1?

Answer 7: Amost the full 110V. The 0.5Ω drops very little voltage along the way.

Question 8: What is the current seen by the body in question 7?

Answer 8:

$$V = IR$$
$$V = I(R_{\text{int}} + R_{\text{body}})$$
$$I = \frac{V}{R_{\text{int}} + R_{\text{body}}}$$
$$I = \frac{110}{0.5 + 1100}$$
$$I \approx 0.1A$$

Question 9: Rather than touching the plug directly, hand 1 will touch an insulated wire instead. The wire is made of hard rubber with resistivity $\rho \approx 10^{13}$, length $L = 10^{-2}$, area $A = 10^{-5}$. Find the voltage experienced by the body.

Answer 9: Model the insulated wire as a resistor R_{ins}.

$$R_{\text{ins}} = \frac{\rho L}{A} = \frac{10^{13} * 10^{-2}}{10^{-5}} = 10^{16}\Omega$$

$$V = IR$$
$$V = I(R_{\text{int}} + R_{\text{ins}} + R_{\text{body}})$$
$$I = \frac{V}{R_{\text{int}} + R_{\text{ins}} + R_{\text{body}}}$$
$$I = \frac{110}{0.5 + 10^{16} + 1100}$$
$$I \approx 0A$$
$$V \approx 0V$$

Question 10: What if instead of putting insulation on the wire, rubber shoes are put on both 3 and 4 instead while touching the plug with hand 1?

Answer 10: It works equally well. As with question 9, the effecitve voltage/current is 0, due to the large resistance.

Caveat: the danger with this approach is that if the electrician accidentally touches something connected to the ground with his other hand 2, then he will enjoy a very nice electric experience...

Protip: Do not get out of a car when a power line falls over it. While inside the car you will not experience any currents, but as soon as you try to exit the car, you create a connection between the power line and the ground.

Back to touchscreens

It turns out that with touchscreens, the calculations involved are really calculations for "voltage divider" circuits.

To calculate the position in both dimensions, first apply voltage in the "Y" plate, and measure voltages across X, then flip to apply voltage across the "X" plate and measure Y voltage.

Question: What is the point relative to which these voltages are measured?

Answer: Some defined ground - the same ground for the power supply, digitizer, and plate.

Note: the digitizer should not sink any current - and in practice the digitizer, in fact, does sink a negligible amount of current.

Question: How can multitouch support be implemented?

Answer: We will see how to do multitouch later in this course.

Design tools/equations:

KCL:

$$\sum_{n=1}^{N} i_n = 0$$

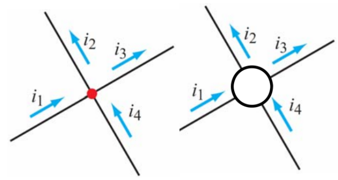


Figure 5: Kirchhoff's current law.

The sum of currents entering a **node** is zero.

KVL:

$$\sum_{n=1}^{N} V_n = 0$$

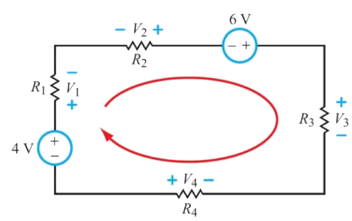


Figure 6: Kirchhoff's voltage law.

The sum of the voltage drops across a **closed loop** is zero.

Question: Consider a battery connected directly to a 1Ω resistor. Use KVL to solve for this circuit.

Answer: Starting from right before the battery, consider a $+12V$ from the battery and a $-12V$ from the resistor. So the total is 0 which is indeed consistent with Kirchoff's voltage law.

Question: How does a battery know how much current to put out?

Answer: The current is set by the battery voltage and the equivalent resistance presented by the load circuit to the battery.

Question: Don't resistors break down?

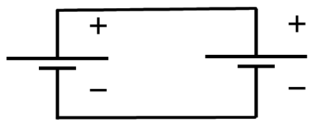
Answer: A large electric field can cause fundamental and irreversible changes (damage) to the material. But for the purposes of this course, resistors will be perfect and will not breakdown.

Question: How do you know what's plus and what's minus in a KVL loop?

Answer: Use the first sign you see. For the graphic example above, starting with the bottom left corner: $-4+V_1-V_2-6+V_3-V_4=0$. Generally speaking, it should follow the passive sign convention to make things easier.

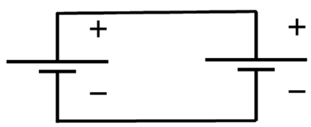
Question: Which is better, KCL or KVL?

Answer: Circuit designers in general tend to favour KCL, but in many cases a mix and match of the two will be required to solve a circuit.

Question: 

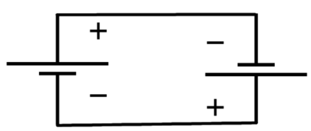
, where the left battery is at 4V and the right battery at 6V?

Answer: Since there is a potential difference between the two batteries, there is a voltage drop across the wire. Hence, there is current flow from the 6V to the 4V battery until the voltages equalize. This will (depending on the characteristics of the battery) charge the 4V battery or cause an explosion.

Question: 

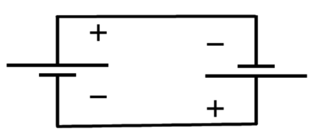
, where the left battery is at 4V and the right battery at 4V?

Answer: Nothing will happen. The two batteries are at equal voltages, so there is no current flow.

Question: 

, where the left battery is at 4V and the right battery at 6V?

Answer: This is equivalent to a short-circuit of a 10 volt battery.

Question: 

, where the left battery is at 4V and the right battery at 4V?

Answer: This is equivalent to a short-circuit of a 8 volt battery.

Example: Series resistors using KCL/KVL.

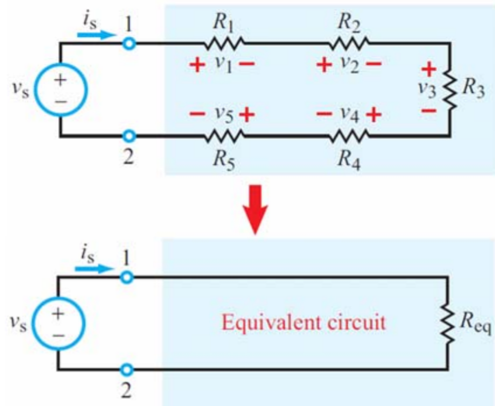


Figure 7: A series circuit.

Using KVL:

Top circuit

$$i_s R_1 + i_s + R_2 + i_s R_3 + i_s R_4 + i_s R_5 - V_s = 0$$
$$R_1 + R_2 + R_3 + R_4 + R_5 = \frac{V_s}{i_s}$$

Bottom circuit

$$i_s R_{\text{eq}} - V_s = 0$$
$$R_{\text{eq}} = \frac{V_s}{i_s}$$

Therefore $R_{\text{eq}} = R_1 + R_2 + R_3 + R_4 + R_5$. ■

Example: Parallel resistors using KCL/KVL.

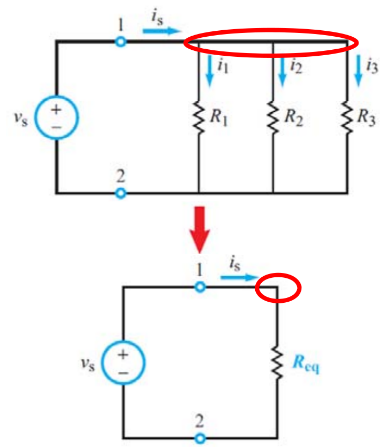


Figure 8: A series circuit.

Using KCL:

Top circuit

$$i_s = i_1 + i_2 + i_3$$
$$i_s = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3}$$
$$i_s = V_s(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3})$$

Bottom circuit

$$i_s = \frac{V_s}{R_{\text{eq}}}$$

Therefore $R_{\text{eq}} = (\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3})^{-1}$. ■

These examples demonstrate a formalized way to approach these circuits.

Passive sign convention

For "passive" elements like resistors and capacitors:

- Current in: $+$
- Current out: $-$

Why? Since resistors can never increase the voltage, it makes sense that the current going into the resistor starts "high" and ends up "low". Imagine the entrance of the resistor being the top of a slide and the other point where current exists being the bottom of a slide.

Touch screen again

Consider again the touch screen, where the top and bottom lines on the plate are highly conductive. All the resistors have the same value (they represent the same size portion of the plate).

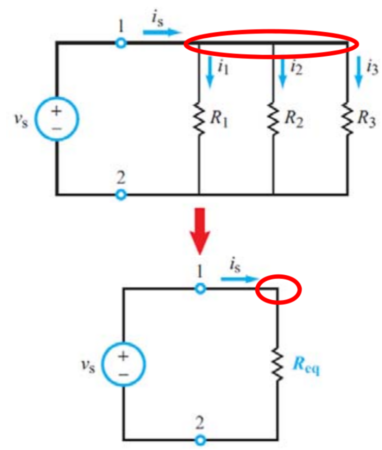


Figure 9: The top and bottom lines on the plate are highly conductive.

The lateral resistors (modelling horizontal connections between parts of the plate) have no voltage drop. Why is this so?

Consider a small KCL which is a smaller version of the top two squares of the big grid:

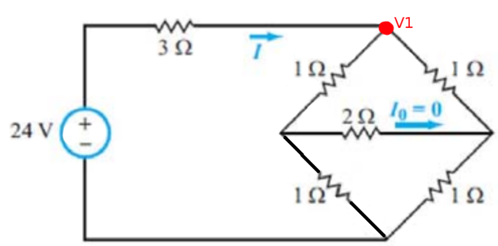


Figure 10: A simplified version of the big touch screen resistor grid circuit.

Consider KCL at the top junction:

$$i = \frac{24 - V_1}{3} - \frac{V_1 - 0}{2} - \frac{V_1 - 0}{2}$$

Question: What happened to the middle resistor?

Answer: It was defined in this problem that the current across that resistor was 0. Nonetheless, even if we calculated the current, because there is no voltage drop across the resistor, there is no current flowing through it, and so it does not have any effect on the rest of the circuit.

Consider the following small circuit:

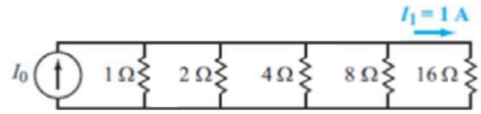


Figure 11: This is a small circuit.

Notice that the voltage across the rightmost 16Ω resistor must be 16 V, since $V = IR$. But notice that the connections across the top form a node. So there $16 = V_1 = V_2 = V_3 = V_4 = V_5 = V$.

Then $I_2 = \frac{V}{R_2} = \frac{16}{8} = 2A$, etc until $I_5 = 16A$.

So the total current, by KCL, must be $I_0 = I_1 + I_2 + I_3 + I_4 + I_5 = (1+2+4+8+16)A = 31A$

In general:

- Everywhere there are unknown voltages/currents across branches/nodes, KCL/KVL must be used.
- KCL is for unknown voltages.
- KVL is for unknown currents/loops.

The last of the touch screens

Here is the touch screen position measurement circuit, in its full glory:

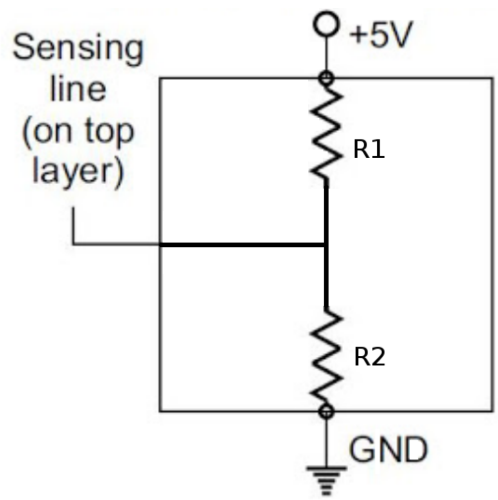


Figure 12: Measurement circuit.

By KVL:

$$0 = 5 - IR_1 - IR_2$$

Notice that the assumption here is that the sensing line has no current - which in the real world is a relatively accurate assumption. (The input resistance of the circuit connected to the sensing line is on the order of $10^8\Omega$ which makes for a negligible current.)

$$I = \frac{V_s}{R_1 + R_2}$$
$$\Delta V_2 = IR_2 = \frac{V_s R_2}{R_1 + R_2}$$

The digitizer will read the quantity ΔV_2 which is indeed correlated with R_2 which is also correlated with where the touch occurred. ■

Relating it to position

Two measurements are required:

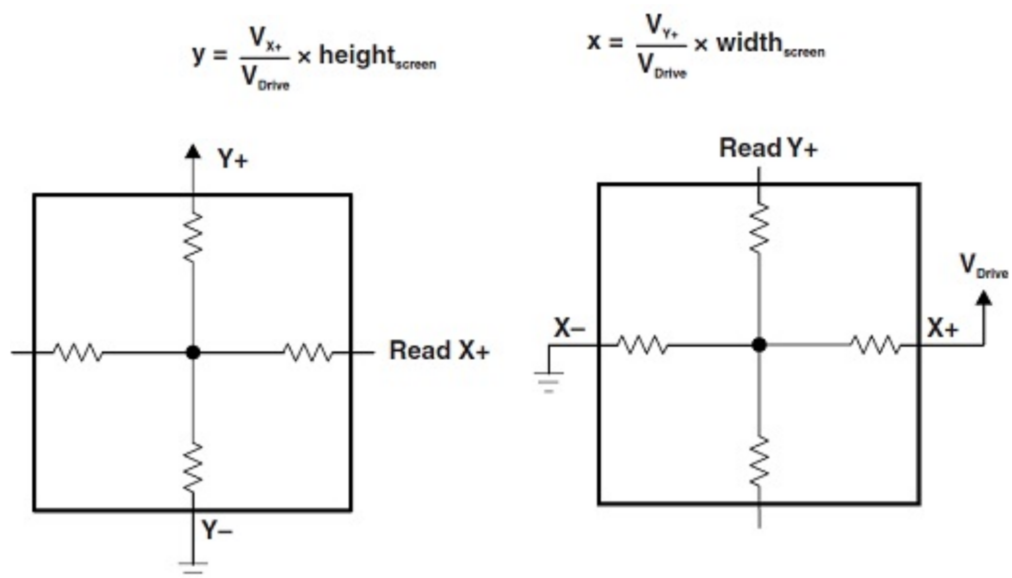


Figure 13: Switching the power supplies in the circuit to read both dimensions.

Question 1: Are two separate power supplies required?

Answer 1: The same power supply is fine. Just have a switch to switch the connections. In the resistor lab, this will be done by hand. In the capacitance lab, this will be done by the Arduino, because it is much faster.

Example: A touch screen has a resistance/length of $1\frac{k\Omega}{\text{cm}}$. The screen has a width of 40 cm, a height of 30 cm, and $V_{\text{applied}} = 5V$.

Question 2: What is V_x, V_y for a touch at the center (horizontal and vertical) of the screen? For the horizontal plate, what are the resistances to the left and to the right of the touch point?

Answer 2: $V_x = 2.5V$, $V_y = 2.5V$. Both resistances to the left and right of the touch point are $20k\Omega$.

Question 3: What is V_x, V_y for a touch at the center (horizontal and vertical) of the top left quadrant of the screen?

Answer 3: $V_x = 1.25V$, $V_y = 3.75V$