

Lecture notes by Ankit Mathur and Nikhil Sathe (3/3/15)

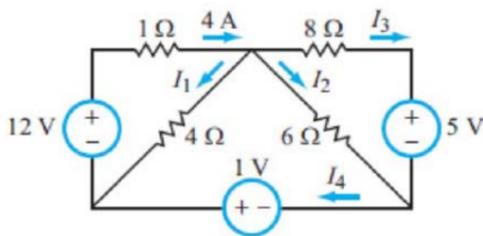
Circuits Review

At a basic level, circuits take advantage of energy and use it to do some work. We can begin a discussion about circuits by abstracting away what looks like a lot of complexity. Consider the black box here



On the left, there are inputs that can be in the form of voltage and/or currents, and, on the right, there is an output created that will also be in terms of the voltage and current that were put into the black box. To understand a circuit, we have to understand the relationships between our inputs and outputs, and we can always define those in terms of current and voltage. In a practical sense, these inputs could be anything - electrical signals within a cell phone, physical signals like the action of touch a screen, etc.

The following exercise helps demonstrate that we can solve the same problem with Kirkhoff's Current Law (KCL) and Kirkhoff's Voltage Law (KVL). Essentially, we are looking for enough equations to solve for the unknowns in the circuit.



Let's say that we want to find the voltage at the node where all 3 triangles that compose the circuit intersect (where the 4 amp current goes into). For example, this can be solved by observing that We know that we have 4 amps coming into the node, and we know that –

$$I_1 + I_2 + I_3 = 4 \tag{1}$$

Now, we could do a KCL on the bottom-left node.

$$I_1 + I_4 = 4 \tag{2}$$

We could do a KVL too, starting the bottom left node and going around the left triangle in the circuit.

$$-12V + (4A)(1\Omega) + (4\Omega)I_1 = 0 \quad (3)$$

We need 1 more equation, since we have 4 unknowns, so we can use a KVL along the middle triangle going clockwise.

$$(6\Omega)I_2 - 1V - (4\Omega)I_1 = 0 \quad (4)$$

Now that we have 4 equations, we can solve for the unknowns. First, we can use equation 3 to solve for $I_1 = 2A$. Then, plug I_1 into equation 4 to get $I_2 = 2A$. Then, plug I_1 into equation 2 to get $I_4 = 2A$. Finally, plug I_1 and I_2 into equation 1 to get $I_3 = 0.5A$.

Even if we are not given that the current going into the top-middle node (which we can call A from now on), we can still solve. Now, instead of 4, we have I_0 there. We then have

$$I_0 = I_1 + I_2 + I_3$$

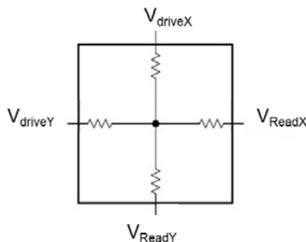
Then, using Ohm's law, we know that $I_0 = \frac{12V - V_A}{1\Omega}$ because the V in the equation is just the voltage difference across the resistor, which has the 12 V battery on one side and an unknown voltage on the other. This process can be repeated for any resistor and for different current variables until we have another system of equations that we are capable of solving.

Side Notes:

1. Even if you don't have enough information about the sources, you can still go ahead and solve the system because we know that we are dealing with constant voltage sources.
2. Nodal technique - you could write a full set of equations for every node (you actually only require all of the nodes but 1, with that one node that's left out being the point of reference for the voltage), and you could find all the equations and solve for the unknowns. This approach would work for any arbitrarily complex circuit, and might be how one would think about making a computer approach this problem.

MultiTouch in Resistive

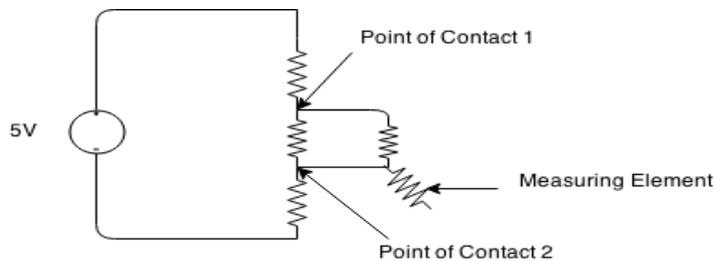
In our original setup, we could not detect x and y simultaneously. Let's prove to ourselves that we actually could not do it. The following is a potential diagram of such a system which drives voltage in both directions.



Let's say we run $V_{driveX} = 3V$ and $V_{driveY} = 3V$ (both of these are in relation to an arbitrary "ground"). Remembering that all voltages are relative, we should notice that none of the resistors in the touchscreen have

any voltage dropped across them - everything is at 3V relative to the external ground. This means that no matter where we touch, we will read out the same voltage (relative to ground).

Now, let's pretend that there are multiple touches on the circuit at once. Then, the issue becomes that the whole reason our circuit worked previously was that the plate used for detection had no current flowing through it, and therefore, the resistance that was associated with that part of the circuit was irrelevant. But, with 2 contacts to the original circuit, there is now current running through both plates. A diagram of such a circuit is below.



In comparison to the original (single-touch) circuit, the readout circuit will measure a single voltage that depends on both touch locations in a more complex way. With a single measurement there is hence no way to figure out where the two touch locations are.

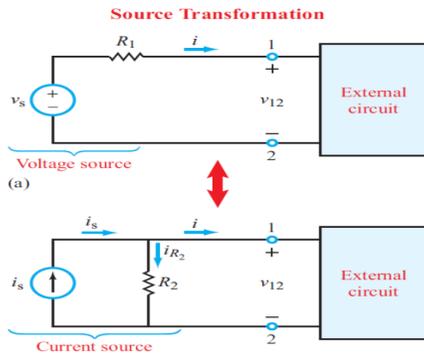
Current Sources and Realistic Sources

A **current source** fixes the current flowing through it rather than a voltage. For example, a solar panel is something that, when shined upon with a fixed amount of photons, roughly generates a fixed current.

Our existing model for a voltage source is ideal, but that is not always the case in real life. A realistic source is an ideal voltage with a resistor in series. Essentially, there is some voltage lost in the series resistance. This is why realistic sources “droop” when the load is increased (e.g. car headlights). That is representative of the current dependence of the voltage source.

Current sources are not ideal either. We are supposed to get a fixed current regardless of what voltage it drives. In reality, there is an equivalent to what happens with voltage sources. The non-ideality is represented as a resistor in parallel, to show that some current is lost. This resistor effectively puts an upper limit on how high the voltage can go.

Any time that we have a voltage with a resistor in series, it can be modeled as a current source with a resistor in parallel. This makes sense from the following exercise:



In this case, in the first diagram with the voltage,

$$I = \frac{V_s - V_{12}}{R_1}$$

In the second diagram,

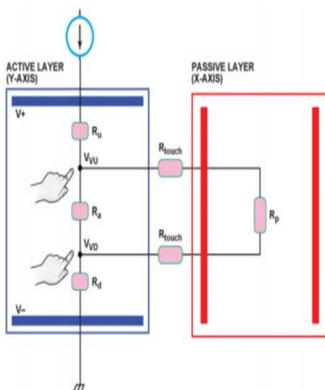
$$I_s = I + I_{R_2}$$

$$I_{R_2} = \frac{V_{12}}{R_2}$$

If we choose $R_2 = R_1$ and $I_s = \frac{V_s}{R_1}$, we find that the current flowing through the external circuit is the same in both cases, and hence the systems are equivalent.

Revisiting Multitouch in Resistive Touchscreens

Now, using a current source, we can actually create a system that allows us to measure multitouch in a resistive system. Now, in this system, when we have multiple touches, we are essentially placing another resistor in parallel with the system, which means that the resistance decreases, and the voltage across the system decreases. This is only possible with a current source because the current is being held constant, which means that the voltage must respond by changing. We can measure that voltage drop to see how far apart the two touches are.



Question: Can this measurement detect a single touch?

No touch:
 $V^+ - V^- = I \cdot (R_u + R_d + R_p)$

2-finger touch:

The passive layer resistance is in parallel with the active layer resistance r_a

Passive layer resistance =
 $R_{touch} + R_p + R_{touch}$

Therefore, we have:

$$V_+ - V_- = I \left(R_u + R_d + R_p \parallel (2R_{touch} + R_p) \right)$$

$$= I \left(R_u + R_d + \frac{R_p (2R_{touch} + R_p)}{R_u + 2R_{touch} + R_p} \right)$$

However, this system cannot really measure a single touch because it is similar to two touches that are incredibly close together.

The remaining portion of multitouch in resistive touchscreens was covered by the next lecture note.