Lecture notes by Mihir Patil (03/20/2015)

# Op Amps with (not really) Infinite Gain

So far we've been assuming that op amps have infinite gain, and it can often be hard to picture how an op amp actually converges to the value between the rails because of this infiniteness. Here we show exactly how the math ends up working out. Remember, our gain is not actually infinite - it is just really, really big.

Recall the following formula from previous lectures for non-inverting amplifiers:

$$V_n = \frac{R_1}{R_1 + R_2} \times V_{\text{out}}$$

Let's define $f$ to be the coefficient in front of $V_{\text{out}}$. Explicitly, $f = \frac{R_1}{R_1+R_2}$.

Now, let's start from the top, with another familiar formula.

$$A(V_p - V_n) = V_{\text{out}}$$

Based on our definition above, we know that this is equivalent to:

$$A(V_p - f \times V_{\text{out}}) = V_{\text{out}}$$

Now, if we simplify this by solving for $V_{\text{out}}$, we get:

$$V_{\text{out}} = \frac{A}{1 + Af} \times V_p$$

Now, as we assume $A$ is really, really large, we can say as $A \to \infty$:

$$V_{\text{out}} = \frac{1}{f} \times V_p$$

So as long as the $A$ (the gain) is really large, it doesn't really matter in our calculations.

## Cross-Correlation

**Cross-correlation** is a measurement of the similarity between two vectors - basically a sliding dot product or inner product.

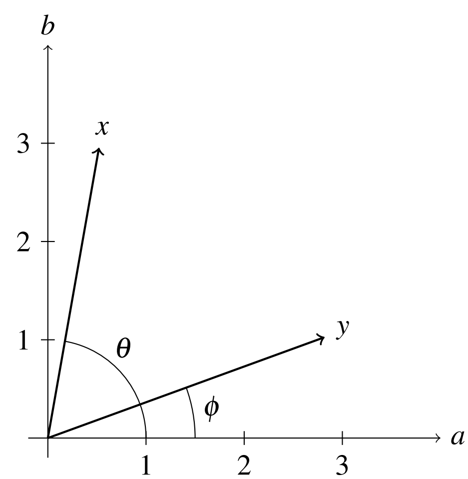
**CDMA** is a signature type for cell phones that consist of positive and negative 1 values. It stands for code division multiple access.

**Cells** are regions of the world where all calls (for a specific company) are routed through a particular cell phone tower. Cells are typically circular in shape, with the center at the place the corresponding cell tower is at.

## Orthogonality

Let's say that $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$. We know from previous encounters with orthogonality that the dot product (inner product) is $x \cdot y = \|x\|\,\|y\|\cos\theta$. Also we know that if this quantity is zero ($x \cdot y = \|x\|\,\|y\|\cos\theta = 0$), then $x$ and $y$ are orthogonal to each other.

Let's try to show this for a case in $\mathbb{R}^2$. Say $x = [x_1 \;\; x_2]^T$ and $y = [y_1 \;\; y_2]^T$. Here's a drawing of them.




Now by definition of the dot product, we know that $x \cdot y = x_1 y_1 + x_2 y_2$.

At this point, we see the following, using high school trigonometry:

$$
\begin{aligned}
x_1 &= \|x\|\cos\phi \\
y_1 &= \|y\|\cos(\phi+\theta) \\
x_2 &= \|x\|\sin\phi \\
y_2 &= \|y\|\sin(\phi+\theta)
\end{aligned}
$$

Now if we take the equalities that we just found and substitute them into the definition of the dot product, then we see that it simplifies down to:

$$x \cdot y = \|x\| \, \|y\| \cos\theta$$

# Examples of Orthogonal Vectors

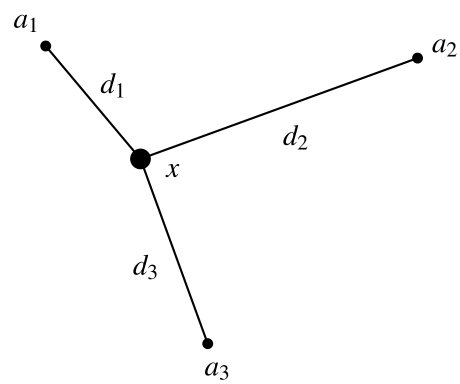
Two orthogonal vectors are shown below. It can be easily shown that their dot product is zero.

$$v_1 = [1 \quad -1 \quad 1 \quad -1]$$
$$v_2 = [1 \quad 1 \quad 1 \quad 1]$$

The reason we can call these CDMA vectors is because they only have values of 1 and $-1$ in them.

# Trilateration

Let's imagine we have a situation like the one below. We know the locations of the beacons $\vec{a}_1, \vec{a}_2, \vec{a}_3$, but don't know the location of the point at $\vec{x}$ (we'll be trying to find out what $\vec{x}$ is). We do know the distances $d_1, d_2, d_3$.




We're trying to find the coordinates of $\vec{x}$ in this diagram.

Now, we know that:

1. $\|\vec{x} - \vec{a}_1\|^2 = d_1^2$
2. $\|\vec{x} - \vec{a}_2\|^2 = d_2^2$
3. $\|\vec{x} - \vec{a}_3\|^2 = d_3^2$

But we have squared terms here. That's not really conducive to our style of computation - we prefer linear terms, so we can use linear algebra.

Let's subtract equation 1 from equation 2, and separately again from equation 3. Then we get:

$$2(\vec{a}_1 - \vec{a}_2)^T \cdot \vec{x} = \|\vec{a}_1\|^2 - \|\vec{a}_2\|^2 - d_1^2 + d_2^2$$

and

$$2(\vec{a}_1 - \vec{a}_3)^T \cdot \vec{x} = \|\vec{a}_1\|^2 - \|\vec{a}_3\|^2 - d_1^2 + d_3^2$$

We can then stick these into a matrix, which will only have linear terms:

$$\begin{bmatrix} 2(\vec{a}_1 - \vec{a}_2)^T \\ 2(\vec{a}_1 - \vec{a}_3)^T \end{bmatrix} \cdot \vec{x} = \begin{bmatrix} \|\vec{a}_1\|^2 - \|\vec{a}_2\|^2 - d_1^2 + d_2^2 \\ \|\vec{a}_1\|^2 - \|\vec{a}_3\|^2 - d_1^2 + d_3^2 \end{bmatrix}$$

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