1. Capacitors and Charge Sharing

(a) Consider the circuit below, with $C_1 = C_2 = 1 \mu F$. Suppose initially $C_1$ is charged to $+1V$, and $C_2$ is charged to $+2V$. How much charge is on $C_1$ and $C_2$? How much energy is stored in each of the capacitors? What is the total stored energy?

\[
\begin{align*}
\phi_1 & \quad + \\
C_1 & \quad | \quad C_2 \\
\quad - & \quad +
\end{align*}
\]

**Answer:** $q_1 = C_1 V_1 = 1 \mu C$. $q_2 = 2 \mu C$. Energy: $E = (1/2)CV^2 = (1/2)Q^2/C = (1/2)QV$, so $E_1 = 1/2 \mu J$, $E_2 = 2 \mu J$, and the total energy is $2.5 \mu J$.

(b) Now the switch is closed (i.e. the capacitors are connected together.) What are the voltages and charges on $C_1$ and $C_2$? What is the total stored energy?

**Answer:** **Charge is always conserved.** $q_1 = q_2 = 1.5 \mu C$. Charge has moved from $C_2$ to $C_1$. This yields a voltage of $1.5V$.

The energies are: $E_1 = 1.125 \mu J$ $E_2 = 1.125 \mu J$. Total energy: $E = 2.25 \mu J$.

(c) Is this more or less energy than before the switch was closed? Why?

**Answer:** Less. Energy is dissipated on the wire. This can be seen if you model the wire as a resistor.

**Note:** the physics behind heat dissipation and the resistor model of the wire are out-of-scope. However, the calculation and identification thereof are in-scope.
(d) Consider the following circuit, with $C_1 = 1 \mu F$, $C_2 = 3 \mu F$. Suppose both capacitors are initially uncharged (0V).

$$\phi_1$$

1V

$C_1$

$C_2$

What are the voltages across the capacitors after the switch is closed? What are the charges on the capacitors?

**Answer:**

Use effective capacitance:

$$C_{eff} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

The series has $C_{eff} = 3/4 \mu F$. So $Q_{eff} = C_{eff} V = 3/4 \mu C$. Note that this means $3/4 \mu C$ has to flow through both $C_1$ and $C_2$, and thus this is the amount of charge stored on each of them.

The voltage across $C_1$ is $Q_{eff}/C_1 = 3/4 V$, and the voltage across $C_2$ is $Q_{eff}/C_2 = 1/4 V$.

(e) Consider the below circuit, with $C_1 = 1 \mu F, R_1 = 1 k\Omega, R_2 = 1 k\Omega$.

$$\phi_1$$

1V

$R_1$

$C_1$

$R_2$

After the switch is closed, and the circuit is allowed to settle, what is voltage across and current through all circuit elements? Note that "settled" here means that the voltages and currents are no longer changing.

**Answer:** Current through a capacitor is defined as $I = \frac{dV}{dt}$. When the voltage settles, no current can be flowing through it. All current will be flowing through $R_1$ and $R_2$. Thus the voltage of the capacitor will be 0.5V in steady-state.
2. Timer Circuit

As we will soon see, keeping track of the amount of time elapsed between the occurrence of two events can be extremely useful. Therefore, in this problem we will explore the design of a circuit that can produce a periodic voltage waveform, where the period of that waveform will be set by the values we choose for our components. In particular, we want to design a circuit that will output +5V for half of the period, and -5V for the other half of the period - i.e., your circuit should output a square wave with a 50% duty cycle.

In order to realize this circuit, you are allowed to use any combination of the following components:

- Op-amps (as comparators)
- Current Sources
- Capacitors
- Switches
- Batteries (i.e., voltage sources)

If you need some control signals (like those we used in the touchscreen lab from the Launchpad) that drive some switches in order to reset and/or initialize some voltages within your circuit, please feel free to utilize those as well.

(a) Sketch a design for a circuit that achieves the timer functionality described above. Don’t worry about setting the value of the period yet or the values of the any of the components yet - just show a schematic for the circuit.

(Hint: If driven by a fixed current, how does the voltage across a capacitor change over time?)

Answer: Conceptually, if we could make a triangle wave, we can get a square wave out of it by using a comparator. Recalling that a constant current into a capacitor gives a ramp voltage (i.e., one side of a triangle wave), we can start with the following initial implementation:

As shown below, the idea in this circuit is that we use the current sources plus the capacitor to generate a triangle wave on $V_{tri}$ that swings between $-5$V and $+5$V; when $V_{tri}$ hits one of the two limits, we want the sign of both the current flowing into the capacitor and the voltage at $V_{out}$ to flip. Thus, the switches $S_1$ and $S_3$ should be on when $V_{out} = +5$V (to drive $V_{tri}$ towards 5V) and switches $S_2$ and $S_4$ should be on when $V_{out} = -5$V (to drive $V_{tri}$ back towards $-5$V).
(b) Now select component values for your design such that the period of your timer circuit is 100 $\mu$s.

**Answer:** In order to select the component values, we first need to figure out what sets the period of oscillation in our circuit. As described above, what we are looking for then is how long it takes for the triangle wave to swing all the way from one of the rails (e.g., $+5V$) to the other rail (e.g., $-5V$) - the period is then just twice of this time (since we need to go from e.g. $+5V$ to $-5V$ and back to $+5V$).

We therefore just need to know how long it takes for that current $I$ to change the voltage on the capacitor by 10V (i.e., $+5V$ - $(-5V)$).

\[
\frac{I \cdot T}{C} = \Delta V
\]

\[
\frac{T_{\text{per}}}{2} = \frac{10V \cdot C}{I}
\]

Now we can finally choose some specific values. Arbitrarily choosing $C = 100\text{pF}$, then $I = 2020\mu\text{A}$.