1. Design Review

Recall the design process.

**Answer:**

(a) **State the goal.** Declare exactly what it is that you want to do. Describe the inputs and outputs and any necessary interpretations of values, e.g. a voltage corresponding to a maglev's height.

(b) **Describe a possible strategy.** Begin breaking the problem down into solvable steps. Relate pieces of information with the equations you know, and make inferences. Draw a black-box diagram. For coding or multi-phase circuits, outline some pseudocode.

(c) **Implement your strategy.** Whether you are writing code, building a circuit, or designing a data structure, take the necessary tools and make the solution you previously described. Since you already broke it down into solveable steps, you can use your existing knowledge and equations to build and integrate those parts.

(d) **Verify and iterate.** Once your solution is built, test it thoroughly to ensure it satisfies your needs. If some aspect fails the requirements, break down your solution and isolate the issue (your black-box diagram will come in handy). Once you find it, you can approach it with this same design process!

2. Dividers for Days

(a) Solve the following circuit for $V_x$. 

![Circuit Diagram](image-url)
Answer:

\[ V_x = \frac{1}{2} V_1 + \frac{1}{2} V_2 \]

(b) You have access to two voltage sources, \( V_1 \) and \( V_2 \). You can use two resistors (as long as \( 0 \leq R < \infty \)). How would you draw a circuit that gives a voltage \( V_x = \frac{1}{4} V_1 + \frac{2}{3} V_2 \)?

Answer: Approach with superposition; even if you know the voltage summer, make sure you know the analysis with KVL/KCL. Using any nonzero values for \( R \):

\[ R_1 = 2R \quad R_2 = R \]

(c) You have two current sources \( I_1 \) and \( I_2 \). You also have a loading resistor \( R_L = 6k\Omega \). Similar to the first part, you can use whatever resistors you want (as long as they are finite integer values). How would you draw a circuit such that the current running through \( R_L \) is \( I_L = \frac{2}{5} (I_1 + I_2) \)?

Answer: Approach with superposition, think of the two currents as one summed current. Use KCL to determine how to divide the currents.

\[ R_L = 6K\Omega, \quad R_1 = 4K\Omega \]
3. Noise Cancelling Headphones Part 1

Almost everyone has tried "noise cancelling" headphones at some point. The basic goal of a noise cancelling headphones is for the user to hear only the desired audio signal and not any other sounds from external sources. In order to achieve this goal, noise cancelling headphones include at least one microphone that listens to what you might have otherwise heard from external sources, and then feeds a signal in to your speakers that cancels (subtracts out) that externally-generated sound.

**Answer:** There are a lot of different solutions for this problem. This solution is aggressive and minimal, so be patient with your understanding. If your solution solves the same problem, you will receive credit.

(a) Let’s start by looking at the most basic part of the headphones: driving the speaker itself with the audio stream we would like to hear. In our system, the source of the audio comes from a digital-to-analog converter (DAC) that translate digital bits to analog voltages. It can modeled as a voltage source with min/max values of 0 \text{V} and 1 \text{V} and a 50Ω source impedance. The speaker can be modeled as an 8Ω resistor, but in order to produce loud enough sounds and not damage the speaker (driving the speaker with non-zero average voltage can damage the transducer within the speaker), it needs to be driven with a range of −1.5V to 1.5V (relative to the ground connected to the DAC, which is the same ground used throughout the system).

You are provided two voltage sources with values −1.5V and 1.5V, an op-amp, and any resistors you would like. Design a circuit that could drive the speaker while meeting the specifications above.

**Answer:** Let’s go through the design method.

**Step 1:**
The goal of this circuit is to have transform an input that ranges from 0V to 1V into an output that ranges from −1.5V to 1.5V.

**Step 2:**
To achieve the goal, we need 3 things:
1. Shift the signal to center at 0V.
2. Provide gain to the signal to go from a 1V range to a 3V range.
3. Maintain a low output resistance, i.e. not load the input signal.

The order of shifting and amplifying does not really matter, but here we choose shifting before amplifier because we do not have high voltage sources (which are needed if we amplify first). When we shift the input signal, the shifted signal does not have to have the same amplitude because we can adjust the gain of the amplifier accordingly. If we make the amplitude of the shifted input signal \(N\), the block diagram is shown below.
Step 3:
To solve the third point, we can simply use an op-amp to drive the output, so that the input and output are separated. We can use a non-inverting amplifier circuit to provide the gain of more than 1, solving the second point. Thus, we are left with the voltage shifter.

It turns out we can use a voltage summer - after all, shifting a voltage is nothing more than subtracting a fixed value. We already derived that the output of a voltage divider is $kV_1 + (1 - k)V_2$, where $k = \frac{R_2}{R_1 + R_2}$, so this is a matter of picking appropriate values for $V_2$ and $k$ and using $V_1$ as the input. We also realize that there is some gain $k < 1$ applied to the input. Thus, we have to adjust the amplifier gain to $A/k = \frac{3}{k}$ ($A = 3$ since our output range is 3V while our input range is 1V).

Let’s build the voltage shifter. We can only use the $-1.5V$ voltage source for $V_2$ if we want to shift the signal level down, i.e. subtract voltage. We also know that we want the shift to be half of the actual signal, since we want to center the signal around 0. Since $V_1$, the DAC voltage, ranges between 0V and 1V, the median voltage is $\frac{1}{2}V$:

\[
kV_{1,med} + (1-k)V_2 = 0\\
(1-k)V_2 = -k \cdot (V_{1,med})\\
(1-k) \cdot (-\frac{3}{2}) = -k \left(\frac{1}{2}\right)\\
3 - 3k = k\\
k = \frac{3}{4}
\]

Knowing our equation $k = \frac{R_2}{R_1 + R_2}$, we can pick values for the resistors such that the ratio is 3/4. However, remember that our DAC has a source resistance of 50Ω. We can use this resistance as $R_1$ since it is in series with the voltage source. It follows that $R_2 = 150\Omega$. Our voltage shifter circuit is shown below. We label the output as $V_{in}$ to the next stage: the amplifier.

![Voltage Shifter Circuit](image)

Remember that this voltage shifter actually has a gain. Specifically, $V_{in} = kV_{DAC} + (1-k)(-1.5) = 0.75V_{DAC} - 0.375$. Thus, this shifter has a gain of 0.75. Following our discussion before, we need to adjust our amplifier to have a gain of $3/0.75 = 4$ and build the appropriate non-inverting amplifier. Since the gain of a non-inverting amplifier is $\frac{R_2 + R_1}{R_1}$, we can choose $R_1 = 1k\Omega$ and $R_2 = 3k\Omega$. The final circuit is shown below.
(b) Now let's look at implementing the noise cancellation. In this problem, we will assume we do not have access to software and therefore cannot digitally remove the noise (as do most noise cancelling headphones). We will therefore focus on implementing the cancellation physically, which is to directly take the (analog) voltage produced by the microphone and subtract it out from the voltage we feed to the speaker.

Let's assume that the microphone can be modeled as a voltage source with min/max values of 0V and 1V (relative to the DAC’s ground) and a 10kΩ source resistance. However, because the materials in the headphones attenuate some of the external sound, the loudest signals picked up by the microphone should correspond to a voltage range of only $-125\text{mV}$ to $+125\text{mV}$ driven onto the speaker.

Expand the circuit from (a) to take the signal from the microphone and subtract it out from the signal that will be driven on to the speaker. You can use op-amps and resistors to do this, but no new voltage sources (except for the model of the microphone of course). Note however that since our speaker driver now needs to handle both the cancellation and the desired audio signal, you can assume that the supply voltages fed to the op-amp have sufficiently large magnitude to ensure that they never clip (reach the power rails). In other words, you should continue to assume that you have $+/-1.5\text{V}$ voltage sources available to use in the rest of your circuit, but the op-amps are now supplied by a separate set of larger, sufficient voltage sources.

**Answer:** Like part (a), we first need to shift the range of the microphone voltage such that it centers around 0V, apply gain, and then add it to our existing circuit. The most intuitive solution is to use two copies of the circuit in part (a) (though using an inverting amp instead of a non-inverting), then feed both outputs to a buffered summer so we have a small output resistance.

We keep the same circuit from part (a) for the audio. For the mic circuit, we use a similar shift, and its resistor values can be found in a similar process to (a). We are given a source resistance, so we use that again. Centering the mic voltage to 0 again gives us a width of 0.75V. Now however, we want our output range to be only $-125\text{mV}$ to $+125\text{mV}$, i.e. a width of 0.25V, so we need a smaller final gain. Also, for this gain, instead of a noninverting amp, we use an inverting amp. This takes care of the "subtraction" aspect.
We know the gain of the inverting amp is \(-\frac{R_2}{R_1}\). We want this to be \(-\frac{0.25V}{0.75V} = -\frac{1}{3}\). So we set \(R_1 = 3K\Omega, R_2 = 1K\Omega\).

However, there is an issue with the circuit we just build. There will be a current running through \(R_1\). Since we connect the shifting circuit to \(R_1\), the voltage out of voltage shifter will be affect (due to the new current). It is important then to add a voltage buffer at this node to prevent this.

Finally, we need to add the two circuits together. Since we took care of the gains and level shift already, we can directly add these values without any further scaling, i.e. take the exact sum of them. We can use the following "summing amplifier" circuit, which combines a voltage summer and a non-inverting amplifier. We will need the non-inverting gain because a voltage summer always applies a linear combination with a gain on each term of less than 1.

Note that since both of the inputs to this circuit will be driven with op amps, we do not have to worry about loading the previous parts, i.e. affecting their output voltages.

Setting up the equality of what we want and the equation that this circuit implements, we get:

\[
V_{\text{audio}} + V_{\text{cancel}} = \left(\frac{R_4}{R_3 + R_4} V_{\text{audio}} + \frac{R_3}{R_3 + R_4} V_{\text{cancel}}\right) \left(\frac{R_5 + R_6}{R_6}\right)
= \left(\frac{R_4}{R_3 + R_4}\right) \left(\frac{R_5 + R_6}{R_6}\right) V_{\text{audio}} + \left(\frac{R_3}{R_3 + R_4}\right) \left(\frac{R_5 + R_6}{R_6}\right) V_{\text{cancel}}
\]
The most obvious solution to this is $R_3 + R_4 = R_5 + R_6$ and $R_6 = R_3 = R_4 = R_5$. We pick all of the resistors to be 1\(\text{K}\Omega\).

We can also take this time to select the rails for this op amp. We specified that we can use larger rails here to handle the bigger range. The maximum output voltage we can achieve is 1.5V from the audio + 0.125V from the mic = 1.625V, so we need at least that large of a supply on both ends.

Putting this all together we get the following circuit:

![Circuit Diagram]

**For Experts:** Alternatively, let’s (aggressively) try to combine the circuits to just use 1 op-amp. From the non-inverting and inverting amplifiers, we can postulate that connecting a signal to the non-inverting input of an op-amp scales and adds the signal at the output while connecting a signal to the inverting input of an op-amp scales and subtracts the signal at the output. Let’s keep this in mind and consider superposition.

We have to somehow feed this voltage to the inverting input of the op-amp so we can subtract the voltage (just like an inverting amplifier). Let’s sketch this circuit and see if this works. We will add a level-shifted $V_{mic}$ (with TBD shift) to the inverting input. Note that the rails of the op-amp has to be greater than the output voltage range, which in this case is 1.5V + 0.125V = 1.625V (which happens when $V_{DAC} = 1V$ and $V_{mic} = 0V$). Also, since the gain is changed, we will need new resistor values in the non-inverting amp.
Since \( V_n = V_p = V_{in} \), we can write KCL at the inverting input node,
\[
\begin{align*}
\frac{V_{mic} - V_{in}}{R_1} + \frac{-1.5 - V_n}{R_1} + \frac{0 - V_{speaker}}{R_3} + \frac{V_{speaker} - V_{in}}{R_4} &= 0 \\
V_{mic}(g_s) - V_{in}(g_s + g_1 + g_2 + g_3) - 1.5g_3 + V_{speaker}g_2 &= 0
\end{align*}
\]
where \( g_n \) is the conductance associated with \( R_n \) \((g_n = \frac{1}{R_n})\). We already know \( g_s \) Now we have 3 unknowns \((g_1, g_2 \text{ and } g_3)\), so we need 3 equations that describe the circuit (since we are only using resistors, the circuit is linear!). From the desired behavior, we want
\[
\begin{array}{c|c|c}
V_{mic} & V_{in} & V_{speaker} \\
0 & 0.375 & 1.625 \\
1 & 0.375 & 1.375 \\
0.5 & 0 & 0
\end{array}
\]
The first line says when the DAC voltage is the highest and the microphone voltage is lowest, we want the highest output \((1.5V + 0.125V)\). The second line says when the DAC voltage is the highest and the microphone voltage is highest, we want \(1.5V - 0.125V = 0.375V\) in the output. Lastly, if the DAC is in the middle and the microphone voltage is also in the middle, we want \(0V\) in the output. We can now construct a matrix equation from the equation we derived.
\[
\begin{bmatrix}
-0.375 & 1.25 & -1.875 \\
-0.375 & 1 & -1.875 \\
0 & 0 & 1.5
\end{bmatrix}
\begin{bmatrix}
g_1 \\
g_2 \\
g_3
\end{bmatrix}
= \begin{bmatrix}
0.0000375 \\
0.000275 \\
0.00005
\end{bmatrix}
\]
To solve for the conductances, it is important to have an invertible matrix. If it is not invertible, try using different cases. For example, if we just use the both maximum inputs, both middle inputs and both minimum inputs, we will get a matrix that is non-invertible. Taking the inverse and multiplying, we get
\[
\begin{bmatrix}
g_1 \\
g_2 \\
g_3
\end{bmatrix}
= \begin{bmatrix}
0.0011 \\
0.0004 \\
0.0000333...
\end{bmatrix}
\]
and \(R_1 = 6k\Omega\) \(R_2 = 2.5k\Omega\) \(R_3 = 30k\Omega\).

The next step will be to include multiple microphones, similar to the "eliminating the troll audio" problem from a previous homework. You will see this in the next discussion.
4. Midterm 2 Review

The last lecture that is in scope is Lecture 7B, Thursday 3/10. The lecture topics that are in scope are:

- All Midterm 1 topics
- Nullspaces
- KVL + KCL
- Ohm’s Law, Resistor Model
- Series/Parallel Equivalence
- Superposition
- Thevenin/Norton Equivalence
- Power
- Capacitor behavior/equations, Parallel plate model
- Charge sharing
- Op amps as comparators and negative feedback golden rules
- Resistance and Capacitance represented by Physical Structures (e.g. biomolecules)
- Touchscreen circuits (Lab)