EECS 16A Designing Information Devices and Systems I
Spring 2016 Elad Alon, Babak Ayazifar Discussion 12A

1. **Review**

2. **Mechanical Problems**

   In each part, find the eigenspace of M associated with the eigenvalue λ.

   (a) $M = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}, \lambda = 1.$

   (b) $M = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}, \lambda = 9.$

   (c) $M = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \lambda = 3.$

3. **Eigenvalues and Special Matrices - For Visualization** The following parts don't require knowledge about how to find eigenvalues. Answer each part by reasoning about the matrix at hand.

   (a) Does a rotation matrix in $\mathbb{R}^2$ have any eigenvalue $\lambda \in \mathbb{R}$?

   (b) Does a reflection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

   (c) Does a projection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?

   (d) If a matrix M has an eigenvalue 0, what does this say about its nullspace? What does this say about the solution(s) of the system of linear equations $M\vec{x} = \vec{b}$?

4. **Steady State Reservoir Levels** We have 3 reservoirs, A, B and C. The pumps system between the reservoirs is depicted in Figure **??**.

   (a) Write the transition matrix representing the pumps system in the problem.

   (b) Assuming you start the pumps with water levels $A_0 = 129, B_0 = 109, C_0 = 0$ (in kiloliters). What would be the steady state water levels (in kiloliters) according to the pumps system described in the problem?

   Hint: If $\vec{x_{ss}} = \begin{bmatrix} A_{ss} \\ B_{ss} \\ C_{ss} \end{bmatrix}$ is a vector describing the steady state levels of water in the reservoirs (in kiloliters), what happens if you fill the reservoirs A, B and C with A_{ss}, B_{ss} and C_{ss} kiloliters of water, respectively and apply the pumps once?

   Hint II: Note that the pumps system preserves the total amount of water in the reservoirs. That is, no water is lost or gained by applying the pumps.

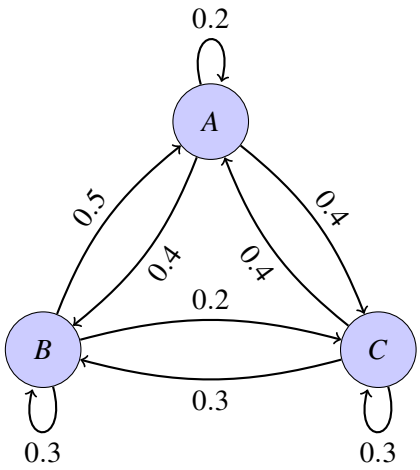

Figure 1: Reservoir pumps system