

### 1. Exploring Nullspaces

- (a) The **column space** of a matrix is the **range** or possible outputs of a transformation/linear operation/function. It is also the **span** of the vectors that form the columns of the matrix.
- (b) The **nullspace** is the set of input vectors that output a zero vector

For the following five matrices, answer the following questions:

- (a) What is the column span of A? What is its dimension?
- (b) What is the nullspace of A? What is its dimension?
- (c) (optional) Do the columns of A form a basis of  $\mathbb{R}^2$ ? Why or why not?

(a)  $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(b)  $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

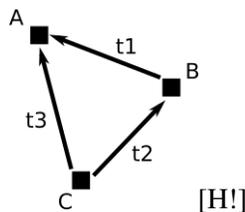
(c)  $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$

(d)  $\begin{bmatrix} -2 & 4 \\ 3 & -6 \end{bmatrix}$

(e)  $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 0 & -1 & -2 \end{bmatrix}$

**2. Traffic Flows** Let's go through the first few parts of HW3 traffic flows problem. Suppose your goal is to measure flow rates of vehicles along roads in Berkeley. However, the city's limited budget prohibits you from placing a traffic sensor along every road. Fortunately, you realize that there is a specific constraint that traffic must obey: the number of cars entering an intersection must equal the number of cars exiting the intersection. We'll see how this constraint helps us determine how many sensors you need for a given set of roads, and where we should place them.

- (a) Begin with a loop of road with three intersections, A, B, and C.  $t_1$  cars flow from B to A per hour.  $t_2$  cars flow from C to B per hour. And  $t_3$  cars flow from C to A per hour.



Because we have determined that the number of cars in the network is conserved, the total number of cars per hour flowing into each node is zero. For example, at node  $B$ ,  $t_2 - t_1 = 0$ . Let's write this constraint as a system of linear equations. We can represent the flows on each road as a vector  $\vec{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$ .

Find the matrix  $G$  such that the equation

$$\begin{bmatrix} & & \\ & G & \\ & & \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

represents the constraint that the sum of flows into each node is zero. This matrix is called the *incidence matrix*. What does each row of this matrix represent? What does each column of this matrix represent?

- (b) We can place sensors on a road to measure the flow through it. But, as we mentioned earlier, the budget is limited. Our goal is to figure out the minimum number of sensors needed to measure flow along every road.

Suppose for the network above we have one sensor reading,  $t_1 = 10$ . Is it possible to calculate the flow rates  $t_2$  and  $t_3$ ?

### 3. Midterm 1 Review

The last lecture that is in scope is Lecture 3A, Tuesday 2/9. The lecture topics that are in scope are:

- Gaussian Elimination
- Imaging/Tomography + Lab
- Linear (in)dependence
- Simple derivations
- Matrix transformations
  - State Transition Matrices
  - Rotations and Reflections
- Span, Range, Column Space
- Rank, Inverses/Invertibility
- Vector Spaces
- Basis