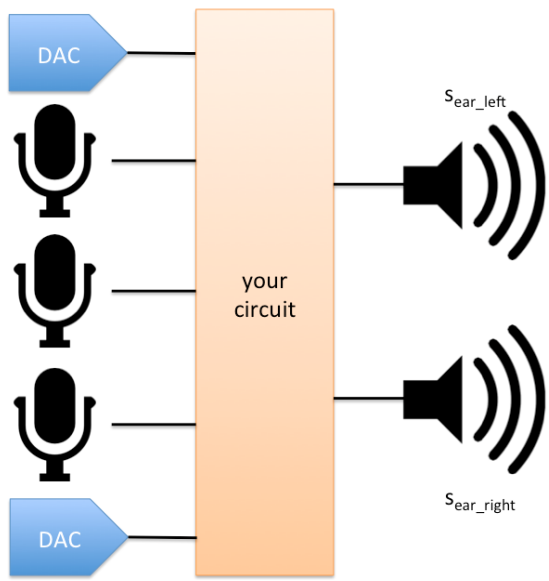
1. Noise Cancelling Headphones Part 2

Almost everyone has tried "noise cancelling" headphones at some point. The basic goal of a noise cancelling headphones is for the user to hear only the desired audio signal and not any other sounds from external sources. In order to achieve this goal, noise cancelling headphones include at least one microphone that listens to what you might have otherwise heard from external sources, and then feeds a signal in to your speakers that cancels (subtracts out) that externally-generated sound.

(a) In the previous part, we had just one speaker and one microphone, but almost all headphones today have two speakers (one for each ear). Adding an extra speaker that can be driven by a separate audio stream typically makes things sound more real to us. For similar reasons, having multiple microphones to pick up ambient sounds from multiple different locations can help us do a better job of cancellation, if we can use that information in the right way.



Let's now assume that our system has 3 microphones and 2 speakers, and that the source of our audio is stereo - i.e., we have two different audio streams s_{left} and s_{right} (produced by two different DACs) that represent the ideal sounds we would like the user to hear in their left and right ear. We have three microphone audio signals s_{mic1}, s_{mic2}, and s_{mic3}, and let's assume that without any active noise cancellation, some fraction of the signal picked up by each microphone would be heard by the user in each of their ears. For example, a_{1left} would represent the fraction of the signal picked up by microphone 1 that will be heard in the user's left ear, a_{2right} would represent the fraction of the signal picked up by microphone 2 that will be in the user's right ear, etc.

Still assuming no noise cancellation and assuming that the DAC/driver circuitry is ideal in producing s_{left} and s_{right}, write a matrix-vector equation you could use to calculate the audio signals s_{ear_left} and s_{ear_right} heard by each of the user's ears.

(b) We define the matrix operation A to relate the signals picked up by each of the microphones to the signals heard by each ear. What matrix B should the active noise cancellation circuitry be aiming to implement in order to ensure that the user doesn't hear any of the sounds picked up by the microphones?

(c) Using resistors and op-amps, and assuming that the microphones can be modeled as voltage sources with a source resistance of $1k\Omega$ and whose value v_{micn} is proportional to s_{micn}, design and sketch a circuit that would implement the cancellation matrix B. You should assume that this circuit has three voltage inputs v_{mic1}, v_{mic2}, and v_{mic3} and two voltage outputs v_{cancel_left} and v_{cancel_right} (corresponding to the voltages that will be subtracted from the desired audio streams in order to cancel the externally-produced sounds). In order to simplify the problem, you can assume that all of the v_{mic} voltages are already centered at $0V$ (relative to the DAC ground).

(d) **BONUS**: Building upon your solutions to all previous parts, and otherwise making the same assumptions about the relative voltage ranges of v_{mic1}, v_{mic2}, and v_{mic3} and available supply voltages, sketch the complete circuit you would use to create the stereo audio on the two speakers while cancelling the noise picked up by the three microphones.