EECS 16A Designing Information Devices and Systems I
Spring 2016 Official Lecture Notes **Note 9**

Introduction

To help you build the intuition behind what you will be learning in the next module on circuits, we are going to start off by taking you on a whimsical journey to a magical universe!

Prof Alon is walking down Telegraph avenue on a cloudy Thursday afternoon. He is leaving Sahai's Optimal Smoothies when he is brutally accosted by a balding old man in wizard robes. The man claims that he has come through time and space to enlist Prof. Alon's help to save his universe. He begins ranting about faeries, imps, and magic. In his fervor, he drops to the ground and begins drawing symbols and diagrams on the sidewalk. From the old man's ramblings and diagrams, Prof. Alon realizes the problems the old man is trying solve can be tackled by his knowledge, and having been convinced that this man is not just a D&D fanatic who ate the wrong breakfast, Prof. Alon decides to help him.

Faerie Mazes

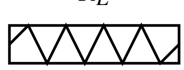
In the old man's universe, humanity lives in harmony with the magical light faeries. The faeries have a magical potential called **vigor**. The vigor is used to control the faeries' mortal enemies, the evil **imps**. After many generations at war, the faeries were able to contain the evil imps in a set of complex **mazes**. These mazes are constructed so that the imps are completely trapped inside them and can never escape.

Imps are imbibed with faerie vigor, which causes them to move through the maze. They also try to travel on paths so that they lose vigor. The maze are designed so that the faeries supply enough vigor to ensure that the imps travel at a minimum flow rate. If the imps were ever to travel slower than the minimum speed, they would begin to accumulate in one place, and then they may be able to escape and wreak havoc on the universe.

The mazes were originally constructed only out of **ideal channels**. Once an imp is moving inside of an ideal channel, it has no choice but to continue doing so. Imps cannot release any vigor by travelling through ideal channels, which made them overly energetic.

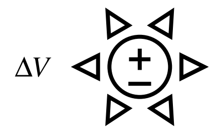
However, over the centuries, the imps figured out that they could convert some of the channels in to **kinked roads** by hitting the walls of the channels. Inside of these kinked roads, imps are able to release some of the vigor applied to them by the faeries; the more vigor the imps are able to release. In particular, the vigor lost, which we denote as ΔV, across a kinked road (R) of length L follows the relationship $\Delta V = I \times R_L$, where I is the number of imps flowing through the road per second. Here is an example of a kinked road:

R_L



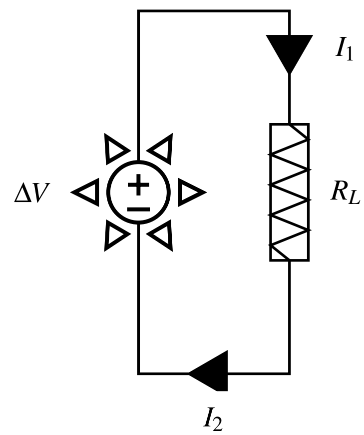
Every maze includes one or more **vigor sources**. Imps crossing through the vigor source in a specific direction have some known amount of additional vigor applied to them. Restored to a higher vigor, they thus are forced to move through the rest of the maze before eventually returning to the vigor source (which will force them to continue back through the maze once again). The strength of the vigor source is a key design choice to ensure that the imps keep moving without using excessive vigor.

We will represent a vigor source as follows:



When imps go from the negative end to the positive end of the following vigor source, imps gain ΔV amount of vigor. The unit of ΔV is vigor.

For example, a maze could look like:



The dark thin lines represent the ideal channels and I_1 and I_2 are the amount of imps that goes through the ideal channel at the locations of the arrows per second. They have units imps/sec. The direction of the arrow represents the direction of the flow. Because the imps are trapped inside the maze and they are forced to keep moving, I_1 and I_2 must satisfy

$$I_1 = I_2.$$

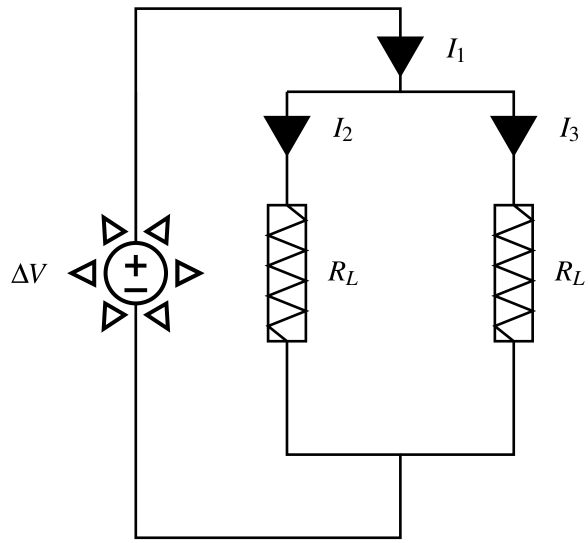
When imps go through the vigor source, they gain ΔV amount of vigor. The imps flow adjusts itself until it no longer changes with time. In order for this to happen, the imps must lose all the additional vigor gained by going through the vigor source before it returns back to the vigor source. Since imps lose $I_1 R_L$ (or $I_2 R_L$) amount of vigor going through the kinked roads, the vigor of the imps must satisfy

$$\Delta V = I_1 R_L = I_2 R_L.$$

Occasionally, to prevent too many imps from going through one channel, the faeries may put a fork in the maze to form **junctions**, so that the imps have to split up along multiple paths. By conservation, any imp that flows into a junction must flow out. No imps accountably enter the system, and no imps leave the system. As before, the imps also cannot build up in one place, lest they break free.

With the path is split, imps have to go through both paths. This is because if too many imps go down one path, too much vigor is lost on the path. The vigor source will not be able to restore the imps to the state of highest vigor. Since this is now a place of lower vigor than the junction itself, the imps will start to flow backwards, implicitly resulting in imps flowing through the other channel. Over time, the flow balances so that the change in vigor across both paths is the same.

For example, in the following maze, the imps are split at the junction.



We see that in this maze, the imps flow I_1 is split into two imps flows I_2 and I_3 at the junction. Since the imps keep flowing and are constrained within the maze, the imps flowing into the junction must be equal to the total imps flowing out of the junction. We thus must have

$$I_1 = I_2 + I_3.$$

In addition, because the imps flows balance themselves so that the change in vigor is the same for the two diverging imps flows, the amount of vigor the imps flows lost by going through the kinked roads is the same on both sides. We know that the drop in vigor for imps flows on the left and right are $I_2 R_L$ and $I_3 R_L$ respectively, so we have the following relationship

$$I_2 R_L = I_3 R_L.$$