EECS 16A Designing Information Devices and Systems I
Spring 2019 Discussion 12B

1. Least Squares: A Toy Example

Let's start off by solving a little example of least squares.

We're given the following system of equations:

$$\begin{bmatrix} 1 & 4 \\ 3 & 8 \\ 5 & 16 \end{bmatrix} \vec{x} = \begin{bmatrix} 3 \\ 1 \\ 9 \end{bmatrix},$$

where $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

(a) Why can we not solve for $\vec{x}$ exactly?

(b) Find $\vec{\hat{x}}$, the *least squares estimate* of $\vec{x}$, using the formula we derived in lecture.

Reminder: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

2. Linearizing Different Problems

Notice that least squares can only be applied to linear systems. Suppose that we have a vector $\vec{x}$ and a vector $\vec{y}$, and $\vec{y}[n] = f(\vec{x}[n])$. We would like to approximate f using least squares, where f is not necessarily a linear function.

(a) Let's begin with a linear approximation. We want to find some a such that $y = ax$. Set this up as a least squares problem. What are the elements in the matrix $\mathbf{A}$?

(b) Let's add a constant to the problem. Suppose that $y = ax + b$. Set this up as a least squares problem. What are the elements in the matrix $\mathbf{A}$?

(c) Suppose that $y = ax^2 + bx + c$. Set this up as a least squares problem. What are the elements in the matrix $\mathbf{A}$?

(d) Suppose that $y = ae^{bx}$. Set this up as a least squares problem. What are the elements in the matrix $\mathbf{A}$?

3. Polynomial Fitting

Let's try an example. Say we know that the output, y, is a quartic polynomial in x. This means that we know that y and x are related as follows:

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$$

We're also given the following observations:

x	y
0.0	24.0
0.5	6.61
1.0	0.0
1.5	−0.95
2.0	0.07
2.5	0.73
3.0	−0.12
3.5	−0.83
4.0	−0.04
4.5	6.42

(a) What are the unknowns in this question? What are we trying to solve for?

(b) Can you write an equation corresponding to the first observation (x_0, y_0), in terms of a_0, a_1, a_2, a_3, and a_4? What does this equation look like? Is it linear?

(c) Now, write a system of equations in terms of a_0, a_1, a_2, a_3, and a_4 using *all of the observations*.

(d) Finally, solve for a_0, a_1, a_2, a_3, and a_4 using IPython. You have now found the quartic polynomial that best fits the data!

