

---

EECS 16A    Designing Information Devices and Systems I    Discussion 2A  
 Spring 2019

---

### 1. Matrix Multiplication

Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix} \quad \mathbf{E} = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix}$$

For each matrix multiplication problem, if the product exists, find the product by hand. Otherwise, explain why the product does not exist.

- (a) **AB**
- (b) **BA**
- (c) **CD**
- (d) **DC**
- (e) **EF**
- (f) **FE**

### 2. Visualizing Matrices as Operations

This problem is going to help you visualize matrices as operations. For example, when we multiply a vector by a “rotation matrix,” we will see it “rotate” in the true sense here. Similarly, when we multiply a vector by a “reflection matrix,” we will see it be “reflected.” The way we will see this is by applying the operation to all the vertices of a polygon and seeing how the polygon changes.

Your TA will now show you how a unit square can be rotated, scaled, or reflected using matrices!

#### Part 1: Rotation Matrices as Rotations

- (a) We are given matrices  $\mathbf{T}_1$  and  $\mathbf{T}_2$ , and we are told that they will rotate the unit square by  $15^\circ$  and  $30^\circ$ , respectively. Design a procedure to rotate the unit square by  $45^\circ$  using only  $\mathbf{T}_1$  and  $\mathbf{T}_2$ , and plot the result in the IPython notebook. How would you rotate the square by  $60^\circ$ ?
- (b) Try to rotate the unit square by  $60^\circ$  using only one matrix. What does this matrix look like?
- (c)  $\mathbf{T}_1$ ,  $\mathbf{T}_2$ , and the matrix you used in part (b) are called “rotation matrices.” They rotate any vector by an angle  $\theta$ . Show that a rotation matrix has the following form:

$$\mathbf{R} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

where  $\theta$  is the angle of rotation. (*Hint: Use your trigonometric identities!*)

- (d) Now, we want to get back the original unit square from the rotated square in part (b). What matrix should we use to do this? *Don't use inverses!* (**Note:** We do not expect you to know inverses at this point; we will cover them soon.)

- (e) Use part (d) to obtain the “inverse” rotation matrix for a matrix that rotates a vector by  $\theta$ . Multiply the inverse rotation matrix with the rotation matrix and vice-versa. What do you get?
- (f) What are the matrices that reflect a vector about the (i)  $x$ -axis, (ii)  $y$ -axis, and (iii)  $x = y$

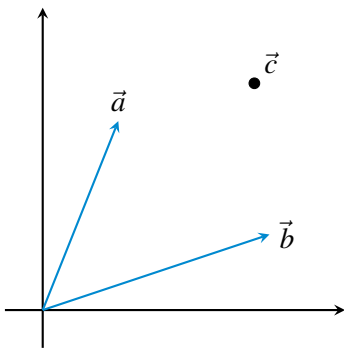
### [Practice Problem] Part 2: Commutativity of Operations

A natural question to ask is the following: Does the *order* in which you apply these operations matter? Follow your TA to obtain the answers to the following questions!

- (a) Let’s see what happens to the unit square when we rotate the square by  $60^\circ$  and then reflect it along the  $y$ -axis.
- (b) Now, let’s see what happens to the unit square when we first reflect the square along the  $y$ -axis and then rotate it by  $60^\circ$ .
- (c) Try to do steps (a) and (b) by multiplying the reflection and rotation matrices together (in the correct order for each case). What does this tell you?
- (d) If you reflected the unit square twice (along any pair of axes), do you think the order in which you applied the reflections would matter? Why/why not?

### 3. Visualizing Span

We are given a point  $\vec{c}$  that we want to get to, but we can only move in two directions:  $\vec{a}$  and  $\vec{b}$ . We know that to get to  $\vec{c}$ , we can travel along  $\vec{a}$  for some amount  $\alpha$ , then change direction, and travel along  $\vec{b}$  for some amount  $\beta$ . We want to find these two scalars  $\alpha$  and  $\beta$ , such that we reach point  $\vec{c}$ . That is,  $\alpha\vec{a} + \beta\vec{b} = \vec{c}$ .



- (a) First, consider the case where  $\vec{a} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $\vec{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ , and  $\vec{c} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ . Draw these vectors on a sheet of paper. Now find the two scalars  $\alpha$  and  $\beta$ , such that we reach point  $\vec{c}$ . What are these scalars if we use  $\vec{a} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$  and  $\vec{b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$  instead?
- (b) Now formulate the general problem as a system of linear equations and write it in matrix form.