1. Dividers for Days

(a) Solve the following circuit for \( v_x \).

\[
\begin{align*}
&\quad R \quad R \\
&V_1 \quad V_2 \\
&\quad \quad \quad \quad v_x \\
&\quad \quad \quad \quad \quad \quad + \\
&\quad \quad \quad \quad \quad \quad - \\
\end{align*}
\]

**Answer:**

\[ v_x = \frac{1}{2} V_1 + \frac{1}{2} V_2 \]

(b) You have access to two voltage sources, \( V_1 \) and \( V_2 \). You can use two resistors (as long as \( 0 \leq R < \infty \)). How would you design a circuit that produces a voltage \( v_x = \frac{1}{3} V_1 + \frac{2}{3} V_2 \)?

**Answer:**

Use superposition. Even if you know the voltage summer, make sure you know the analysis with KVL/KCL. Using any nonzero values for \( R \):

\[
\begin{align*}
R_1 &= 2R \\
R_2 &= R \\
\end{align*}
\]

\[
\begin{align*}
&\quad R_1 \quad R_2 \\
&V_1 \quad V_2 \\
&\quad \quad \quad \quad v_x \\
&\quad \quad \quad \quad \quad \quad + \\
&\quad \quad \quad \quad \quad \quad - \\
\end{align*}
\]

(c) You have two current sources \( I_1 \) and \( I_2 \). You also have a load resistor \( R_L = 6k\Omega \). Similar to the first part, you can use whatever resistors you want (as long as they are finite integer values). How would you design a circuit such that the current running through \( R_L \) is \( I_L = \frac{2}{5}(I_1 + I_2) \)?

**Answer:**

Use superposition, so think of the two currents as one summed current. Use KCL to determine how to divide the currents.
2. **Baking**

You decide to make a little oven (from scratch) for a party so the kids can bake things. In order to do this you will need a filament to radiate heat inside the oven. A filament can be modeled as a resistor with all the power dissipated by the resistor being converted into heat.

You find a 1 m-long strip of some filament material that has a resistance of 20Ω and you want to cut a length of it to connect it directly to your 20V battery (which has internal resistance of 1Ω) to heat the play oven. Assume that the resistance of the strip of filament is proportional to its length.

(a) How long should you cut the strip to maximize the amount of heat generated in the oven?

**Answer:**

\[
V_{\text{fil}} = \frac{R_{\text{fil}}}{R_{\text{fil}} + 1 \text{Ω}} \cdot 20 \text{V}
\]

\[
P_{\text{fil}} = \frac{V_{\text{fil}}^2}{R_{\text{fil}}} = \frac{R_{\text{fil}}}{(R_{\text{fil}} + 1 \text{Ω})^2} \cdot (20 \text{V})^2
\]

We want to maximize \(P_{\text{fil}}\) with respect to \(R_{\text{fil}}\), so we compute the derivative of \(P_{\text{fil}}\) with respect to \(R_{\text{fil}}\) using the quotient rule.

\[
\frac{dP_{\text{fil}}}{dR_{\text{fil}}} = \frac{(R_{\text{fil}} + 1 \text{Ω})^2 - 2R_{\text{fil}}(R_{\text{fil}} + 1 \text{Ω})}{(R_{\text{fil}} + 1 \text{Ω})^4} \cdot (20 \text{V})^2 = 0
\]

\[
-R_{\text{fil}} + 1 \text{Ω} = 0
\]

\[
R_{\text{fil}} = 1 \text{Ω}
\]
(b) How much heat (in units of W) is available in the oven?

Answer:

\[
V_{\text{fil}} = \frac{1 \Omega}{1 \Omega + 1 \Omega} \cdot 20 \text{V} = 10 \text{V}
\]

\[
P_{\text{fil}} = \frac{(10 \text{V})^2}{1 \Omega} = 100 \text{W}
\]
3. PetBot Design (Fall 2016 Final)

In this problem, you will design circuits to control PetBot, a simple robot designed to follow light. PetBot measures light using photoresistors. A photoresistor is a light-sensitive resistor. As it is exposed to more light, its resistance decreases. Given below is the circuit symbol for a photoresistor.

![Photoresistor Circuit Symbol](../figures/PetBot1.png)

Below is the basic layout of the PetBot. It has one motor on each wheel. We will model each motor as a 1Ω resistor. When motors have positive voltage across them, they drive forward; when they have negative voltage across them, they drive backward. At zero voltage across the motors, the PetBot stops. The speed of the motor is directly proportional to the magnitude of the motor voltage. The light sensor is mounted to the front of the robot.

(a) **Speed control** – Let us begin by first having PetBot decrease its speed as it drives toward the flashlight. Design a motor driver circuit that outputs a decreasing positive motor voltage as the PetBot drives toward the flashlight. The motor voltage should be at least 5V far away from the flashlight. When far away from the flashlight, the photoresistor value will be 10kΩ and dropping toward 100Ω as it gets closer to the flashlight.

In your design, you may use any number of resistors and op-amps. You also have access to voltage sources of 10V and −10V. Based on your circuit, derive an expression for the motor voltage as a function of the circuit components that you used.

**Answer:**

![Motor Driver Circuit](../figures/PetBot1.png)

The output of the above circuit is:

\[ v_{out} = \frac{R_p}{R_p + R} \times 10V \]

\( R_p \) represents the photoresistor, and \( R \leq 10k\Omega \).

(b) **Distance control** – Let us now have PetBot drive up to a flashlight (or away from the flashlight) and stop at distance of 1m away from the light. At the distance of 1m from the flashlight, the photoresistor has a value 1kΩ.

Design a circuit to output a motor voltage that is positive when the PetBot is at a distance greater than 1m from the flashlight (making the PetBot move toward it), zero at 1m from the flashlight.
(making the PetBot stop), and negative at a distance of less than 1 m from the flashlight (making the PetBot back away from the flashlight).

In your design, you may use any number of resistors and op-amps. You also have access to voltage sources of 10 V and −10 V. Based on your circuit, derive an expression for the motor voltage as a function of the values of circuit components that you used.

**Answer:**

![Circuit Diagram 1](image1)

Alternatively:

![Circuit Diagram 2](image2)