1. Periodic Signals

Periodic signals are ones that repeat themselves entirely after some time period. That is, after some time $p$, the signal $x(n)$ repeats itself so that $x(n + p) = x(n)$. Discrete periodic signals, during the period, do not update continuously through time. They instead update in specific discrete time steps, as if sampling a continuous signal.

Since there are a finite number of "unique" sequences in a discrete periodic signal, it is natural for us to represent the signal as a vector. We observe one period and treat the value at each time step as a different value in our vector.

Let us study the signal $\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ that is periodic over $p = 2$.

(a) Write the signal as a linear combination of the standard/canonical basis. What signals do these vectors correspond to? How can we interpret the linear combination?

**Answer:**

The canonical vectors appear as signals that repeat $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ respectively every 2 time steps. The signal can be decomposed into the "sub"-signals $\begin{bmatrix} 4 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$. The linear combination here is similar to a superposition of waves in real life.
(b) Write the signal as a linear combination of the basis \( \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} \). What signals does these vectors correspond do? How can we interpret the linear combination?

**Answer:**

The signal can be decomposed into the "sub"-signals \( \begin{bmatrix} 3 \\ 3 \end{bmatrix} \) and \( \begin{bmatrix} 1 \\ -1 \end{bmatrix} \). Note how these sub-signals are different; one of the signals is constant, and the other oscillates at a regular frequency.

(c) Project the signal \( \begin{bmatrix} 4 \\ 2 \end{bmatrix} \) onto each of the vectors in the previous part. How does these vectors relate to the linear combination from the previous part?

**Answer:** The projection should the same as the linear combination because the basis is orthogonal.

(d) Given the above, what is an easy way to find the coefficients for describing the signal as a linear combination of our basis? What property must hold about our basis?

**Answer:** If we can find an orthogonal basis, then we can use projections to describe a linear combination in one basis. Then we can do a change of basis to return to our original basis (if necessary for the situation).

2. **Correlation**

You are given the following two signals:

Assume that both signals are periodic with period 5, that is, each plot shows one full period of a periodic signal.
(a) Sketch the autocorrelation (correlation with itself) of signal 1.

**Answer:**

The autocorrelation is as follows. Autocorrelation is a special case of cross-correlation (it is the cross-correlation of a signal with itself). See the answer for part [c] for an example of how to compute cross-correlation.

![Periodic Autocorrelation of Signal 1](image)

(b) Sketch the autocorrelation of signal 2.

**Answer:**

![Periodic Autocorrelation of Signal 2](image)

(c) Sketch the cross-correlation of signal 1 with signal 2. Suppose we know signal 2 is a delayed (and attenuated) version of signal 1. What does the cross-correlation tell us about the delay?

**Answer:**

Represent signal 1 as the vector \( \mathbf{x} = [-4 \quad 8 \quad -4 \quad 0 \quad 0]^T \). We write \( x[0] = -4, x[1] = 8 \), etc. Similarly, represent signal 2 as the vector \( \mathbf{y} = [4 \quad -2 \quad 0 \quad 0 \quad -2]^T \).

To compute the cross-correlation \( \rho_{xy}[m] \), we circularly shift the vector \( \mathbf{y} \), and compute the inner-product of the shifted \( \mathbf{y} \) with the original \( \mathbf{x} \). For example, to compute the cross-correlation at 0 (corresponding to shifting \( \mathbf{y} \) by 0):

\[
\rho_{xy}[0] = \langle \mathbf{x}, \mathbf{y} \rangle = (-4)(4) + (8)(-2) + (-4)(0) + (0)(0) + (0)(-2) = -32
\]

To compute the cross-correlation at lag 1, we first circularly-shift \( \mathbf{y} \) right by 1. Call this shifted version \( \mathbf{y}^{(1)} \)

\[
\mathbf{y} = [4 \quad -2 \quad 0 \quad 0 \quad -2]^T
\]

\[
\mathbf{y}^{(1)} = [-2 \quad 4 \quad -2 \quad 0 \quad 0]^T
\]
Then:
\[ \rho_{xy}[m][1] = \langle \tilde{x}, \tilde{y}^{(1)} \rangle = 48 \]

It can be useful to visualize correlations as “shifting \( y[n] \) along \( x[n] \)” and pointwise-multiplying the two signals. For example, to compute \( \rho_{xy}[1] \) as above:

<table>
<thead>
<tr>
<th>( \tilde{x} )</th>
<th>-4</th>
<th>8</th>
<th>-4</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{y}^{(1)} )</td>
<td>-2</td>
<td>4</td>
<td>-2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \langle \tilde{x}, \tilde{y}^{(1)} \rangle )</td>
<td>8</td>
<td>+ 32</td>
<td>+ 8</td>
<td>+ 0</td>
<td>+ 0</td>
</tr>
</tbody>
</table>

To compute \( \rho_{xy}[2] \), we shift \( \tilde{y} \) by 2:

<table>
<thead>
<tr>
<th>( \tilde{x} )</th>
<th>-4</th>
<th>8</th>
<th>-4</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{y}^{(2)} )</td>
<td>0</td>
<td>-2</td>
<td>4</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>( \langle \tilde{x}, \tilde{y}^{(2)} \rangle )</td>
<td>0</td>
<td>+ -16</td>
<td>+ -16</td>
<td>+ 0</td>
<td>+ 0</td>
</tr>
</tbody>
</table>

Continuing in this way, we find \( \rho_{xy}[m] = [-32 \quad 48 \quad -32 \quad 8 \quad 8]^T \), as shown below.

The peak of this cross-correlation is at lag \( m = 1 \), meaning that the best alignment of signal 2 against signal 1 is when signal 2 is shifted right by \( m = 1 \).

**Remark (optional):** There is a subtlety here: it appears that signal 2 actually *leads* signal 1. That is, signal 2 appears “delayed” by \( -1 \) time steps. However, recall that the signals are periodic with period 5. Thus, the actual delay could have been \( -1 + 5 = 4 \), or \( -1 + 5 \cdot 2 = 9 \), or in general \( -1 + 5k \) for some integer \( k \).

**3. Autocorrelation Peak**

Let \( \rho_{xx}[m] \) be the autocorrelation of an \( N \)-periodic signal \( x[n] \). Prove that \( \rho_{xx}[0] \geq |\rho_{xx}[m]| \) \( \forall m \). In other words, the autocorrelation peak (maximum value of autocorrelation) of any periodic signal always occurs at lag \( m = 0 \).

**Answer:**

**Method 1:** First, we define \( \tilde{x} = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix} \) and \( \tilde{x}^{(m)} = \begin{bmatrix} x[-m] \\ x[-m+1] \\ \vdots \\ x[-m+N-1] \end{bmatrix} \). Recall the geometric inter-
pretation of the inner product $\langle \vec{x}, \vec{y} \rangle = \| \vec{x} \| \| \vec{y} \| \cos \theta$. Applying this, we find that

$$\rho_{xx}[0] = \langle \vec{x}, \vec{x} \rangle = \| \vec{x} \|^2,$$

$$|\rho_{xx}[m]| = \| \vec{x} \| \| \vec{x}(m) \| \cos \theta |$$

$$\leq \| \vec{x} \|^2 = \rho_{xx}[0],$$

where the inequality comes from the fact that $\cos \theta$ is bounded between $-1$ and $1$.

**Method 2:** We make note of the expansion

$$(x[n + m] - x[n])^2 = x^2[n + m] + x^2[n] - 2x[n + m]x[n]$$

and compute

$$\rho_{xx}[m] = \sum_{n=0}^{N-1} x[n + m]x[n]$$

$$= \sum_{n=0}^{N-1} \frac{1}{2} \left( x^2[n] + x^2[n + m] - (x[n + m] - x[n])^2 \right)$$

$$= \frac{1}{2} \left( \sum_{n=0}^{N-1} x^2[n] + \sum_{n=0}^{N-1} x^2[n + m] - \sum_{n=0}^{N-1} (x[n + m] - x[n])^2 \right)$$

$$= \frac{1}{2} \left( \rho_{xx}[0] + \rho_{xx}[0]\right) - \frac{1}{2} \sum_{n=0}^{N-1} (x[n + m] - x[n])^2$$

$$= \rho_{xx}[0] - \frac{1}{2} \sum_{n=0}^{N-1} (x[n + m] - x[n])^2$$

$$\leq \rho_{xx}[0],$$

where the inequality comes from the fact that we are subtracting a sum of squares, which must be non-negative.

**Method 3:** Imagine that $\vec{x}$ is just a list of numbers. Now imagine that you are allowed to rearrange the numbers in $\vec{x}$ to form a new vector $\tilde{\vec{x}}$. How would you rearrange the numbers to maximize $\langle \vec{x}, \tilde{\vec{x}} \rangle$? Because an inner product is just a sum of pairwise products, it is clear that in order to maximize this sum, you want to multiply numbers of the same sign. Furthermore, to make the largest possible contribution to the sum, you want to pair the largest (magnitude) number in $\vec{x}$ with the largest (magnitude) number in $\tilde{\vec{x}}$. Then, you want to pair the second largest with the second largest, the third with the third, and so on. Continuing in this fashion, you find that the $\tilde{\vec{x}}$ that maximizes the inner product is simply $\vec{x}$. Any other arrangement will produce an inner product of lesser or equal absolute value. Because circular shifts are just a special case of rearrangement, we have proved the claim.

4. **Search and Rescue Dogs**

Berkeley’s Puppy Pound needs your help! While Mr. Muffin was being walked, the volunteer let go of his leash and he is now running wild in the streets of Berkeley (which are quite dangerous)! Thankfully, all of the puppies at the pound have a collar that sends a bluetooth signal to receiver towers, which are spread...
throughout the streets (pictured below). If the puppy/collar is within range of the receiver tower, the collar will send the tower a message: the distance of the collar to the tower. Each cell tower has a range of 3 city blocks. Can you help the pound locate their lost puppy?

**Note:** A city block is defined as the middle of an intersection to the middle of an adjacent intersection (scale provided on map.) Mr. Muffin is constrained to running wild in the streets, meaning he won’t be found in any buildings. If your TA asks ‘Where is Mr. Muffin?’ it is sufficient to answer with his intersection or ‘between these two intersections’.

![Dog](http://www.pupsmile.com/wp-content/uploads/2012/11/running_happy_dog-1024x684.jpeg)

(a) You check the logs of the cell towers, and they have received the following messages:

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>1.3</td>
</tr>
<tr>
<td>W</td>
<td>3</td>
</tr>
<tr>
<td>E</td>
<td>1.5</td>
</tr>
<tr>
<td>S</td>
<td>3</td>
</tr>
</tbody>
</table>

On the map provided, identify where Mr. Muffin is!

CAL CAMPUS

![Map](http://www.pupsmile.com/wp-content/uploads/2012/11/running_happy_dog-1024x684.jpeg)

**Note:**

Can you set this up as a system of equations? Is it linear? If it’s not linear, can you think of a way to make it linear? Now, how do you set this up in matrix form?

Hint: Set (0,0) to be Channing and Bowditch.

Hint 2: distance = \( \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \)

Hint 3: You don’t need all 4 equations. You have two unknowns, x and y. You know from lecture that you need three circles to uniquely find a point. How can you use the third circle/equation to get two equations and two unknowns?

Answer: First, set up the system of equations:

\[
\begin{align*}
(x - 0)^2 + (y - 2)^2 &= 1.3^2 \\
(x + 2)^2 + (y - 0)^2 &= 3.0^2 \\
(x - 2)^2 + (y - 0)^2 &= 1.5^2
\end{align*}
\]

Simplify out:

\[
\begin{align*}
x^2 + y^2 - 4y + 4 &= 1.3^2 \\
x^2 + 4x + 4 + y^2 &= 3.0^2 \\
x^2 - 4x + 4 + y^2 &= 1.5^2
\end{align*}
\]

Then subtract equation (1) from equations (2) and (3):

\[
\begin{align*}
4x + 4y &= 3.0^2 - 1.3^2 \\
-4x + 4y &= 1.5^2 - 1.3^2
\end{align*}
\]
This solves to \( x = 0.84 \), \( y = 0.98 \) which is roughly College and Durant.

(c) Suppose Mr. Muffin is moving fast, and by the time you get to destination in part (a) he’s already run off! You check the logs of the cell towers again, and see the following updated messages:

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>2.2</td>
</tr>
<tr>
<td>W</td>
<td>Out of Range</td>
</tr>
<tr>
<td>E</td>
<td>1.1</td>
</tr>
<tr>
<td>S</td>
<td>Out of Range</td>
</tr>
</tbody>
</table>

Can you find Mr. Muffin?

**CAL CAMPUS**

**SOUTH OF DWIGHT**

**Answer:** With two out of range sensors, you might think that you will not be able to find a unique solution (you need 3 circles to intersect at a point.) The trick is that out of range still provide information on where Mr. Muffin is NOT. See the diagram below.
(d) Mr. Muffin is a very mischievous puppy, and while playing and running around he damaged his collar. The transmitter on his collar will still send a signal to the receiver towers, but the distance sensor has noise. You check the logs of the cell towers, and they have received the following messages:

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>1.7 ± 0.5</td>
</tr>
<tr>
<td>W</td>
<td>2.1 ± 0.2</td>
</tr>
<tr>
<td>E</td>
<td>Out of Range</td>
</tr>
<tr>
<td>S</td>
<td>Out of Range</td>
</tr>
</tbody>
</table>

On the map provided, identify where Mr. Muffin is! Can you find exactly where he is?
5. Retail Store Marketing

**Intro**  The retail store EehEeh Sixteen would like to create a smart system where it decides which promotion to give to its customers when they checkout, depending on things they may be interested in. The
The goal

EehEeh Sixteen hired the same intern from the Framingham heart study to devise an algorithm that takes a customer’s purchase subtotals in the four categories listed above (food, movies, art and books & supplies), and decides which promotion to print on the receipt. The intern is lost and given the awesomeness of your help last time, he needs your help again. In this problem, you will walk him through a possible design of such an algorithm.

(a) Assuming we somehow have the interests of a customer \( c \) in a vector \( \vec{x}_c \) and a set of promotions \( A_1, A_2, \ldots, A_N \), with their attached vectors of scores \( \vec{s}_{A_1}, \vec{s}_{A_2}, \ldots, \vec{s}_{A_N} \). We would like to select which promotion is best aligned with the preferences of the customer. Assuming we have a function \( \text{sim}(\vec{x}_c, \vec{s}_A) \) which outputs a similarity score (higher score means more similar) between the customer \( c \) and the promotion \( A \), how can we select which promotion to print to the customer on her receipt?

Answer: We iterate over all the promotions, \( A_1, A_2, \ldots, A_N \): For each promotion \( A \), we calculate \( \text{sim}(\vec{x}_c, \vec{s}_A) \) and we pick the promotion \( A \) with the highest score \( \text{sim}(\vec{x}_c, \vec{s}_A) \).

Note that this problem is a generalization of the smoothies problem from HW1, since it infers the general preferences of a customer and then picks the best promotion tailored to their preferences.

(b) Would \( \text{sim}_1(\vec{x}_c, \vec{s}_A) = \|\vec{x}_c - \vec{s}_A\| \) be a good similarity measure? Why? What about \( \text{sim}_2(\vec{x}_c, \vec{s}_A) = \frac{1}{\|\vec{x}_c - \vec{s}_A\|} \)? Why? What about \( \text{sim}_3(\vec{x}_c, \vec{s}_A) = \langle \vec{x}_c, \vec{s}_A \rangle \)? Why? What about \( \text{sim}_4(\vec{x}_c, \vec{s}_A) = \langle \vec{x}_c, \vec{s}_A \rangle / \|\vec{s}_A\| \)? Why?

Answer:

i. Distance: \( \text{sim}_1(\vec{x}_c, \vec{s}_A) = \|\vec{x}_c - \vec{s}_A\| \) is not good because the farther these vectors are from each other the higher the score. Which is opposite to what we wanted.

ii. Inverse distance: \( \text{sim}_2(\vec{x}_c, \vec{s}_A) = \frac{1}{\|\vec{x}_c - \vec{s}_A\|} \) is a better option. It grows when the preferences are closer and shrinks when the preferences are farther apart.

iii. Inner product: \( \text{sim}_3(\vec{x}_c, \vec{s}_A) = \langle \vec{x}_c, \vec{s}_A \rangle \) is an even better option. Geometrically, it is almost the projection of the customer preferences on the promotion target, which makes sense too. In addition, it
can also provide negative scores, which may indicate that a customer is opposite to the target audience (advantage over inverse distance). The problem with this choice is that it scales with the norm of \( \vec{s}_A \). As a matter of fact, in the example used here, \( \frac{8}{9} = sim_3(\vec{x}_s, \vec{s}_A) > sim_3(\vec{x}_s, \vec{s}_A) = \frac{2}{3} = \frac{6}{9} \) which is not as we want it to be.

iv. Projection: \( sim_4(\vec{x}_s, \vec{s}_A) = \langle \vec{x}_s, \frac{\vec{s}_A}{\|\vec{s}_A\|} \rangle \) is an even better option. It has the goods of \( sim_3 \) and solves the scaling problem. Lock on this choice for the rest of the problem.

(c) The intern hands you research that the EehEeh Sixteen research division conducted, which calculated the distribution of spending in the store for people who are purely interested in only one category. The results are depicted in Table 1. Use this information to devise a system of linear equations, such that solving this system will result in the customer’s preferences given her spending.

<table>
<thead>
<tr>
<th>Spending Category</th>
<th>Food</th>
<th>Movies</th>
<th>Art</th>
<th>Books &amp; Supplies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Party</td>
<td>40%</td>
<td>33%</td>
<td>22%</td>
<td>5%</td>
</tr>
<tr>
<td>Family</td>
<td>70%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>Student</td>
<td>20%</td>
<td>10%</td>
<td>15%</td>
<td>55%</td>
</tr>
<tr>
<td>Office</td>
<td>5%</td>
<td>2%</td>
<td>20%</td>
<td>73%</td>
</tr>
</tbody>
</table>

Table 1: The distribution of spending of people in each category.

**Answer:** First, let’s normalize the spending to get percentages only. This step is important since it will make our algorithm insensitive to the absolute amount of spending. To see this better, the preferences of someone who spends $4 on food, $3.3 on movies, $2.2 on art and $.5 on books & supplies are equal to the preferences of someone who spends $40 on food, $33 on movies, $22 on art and $5 on books & supplies (both are pure budget for party products, according to the research in table 1) and therefore both should have the same preferences scores in the end. For a given customer, let \( T_{food}, T_{movies}, T_{art} \) and \( T_{books} \) represent the customer’s total spending on food, movies, art and books, respectively (which are observed). To get the percentages of spending \( p_{food}, p_{movies}, p_{art} \) and \( p_{books} \), we normalize each by the total spending. E.g., \( p_{food} = \frac{T_{food}}{T_{food} + T_{movies} + T_{art} + T_{books}} \)

The system of linear equations that describes the spending above, assuming the spending are observed:

\[
\begin{align*}
0.4c_{party} + 0.7c_{family} + 0.2c_{student} + 0.05c_{office} &= p_{food} \\
0.33c_{party} + 0.1c_{family} + 0.1c_{student} + 0.02c_{office} &= p_{movies} \\
0.22c_{party} + 0.1c_{family} + 0.15c_{student} + 0.2c_{office} &= p_{art} \\
0.05c_{party} + 0.1c_{family} + 0.55c_{student} + 0.73c_{office} &= p_{books}
\end{align*}
\]

Another reason why normalizing the spending is good: Note that if we sum all equations, we get \( c_{party} + c_{family} + c_{student} + c_{office} = p_{food} + p_{movies} + p_{art} + p_{books} \) which means \( c_{party} + c_{family} + c_{student} + c_{office} = 1 \), which means that the sum of all preferences now have to sum to one (but note that some scores may be negative!)

(d) Combine these results into a complete algorithm.

**Answer:** Use algorithm[1] for reference. You may want to simplify things by using simpler notation.

(e) Run the algorithm on a customer, Jane Doe, that spent $6 on food, $4 on movies, $1 on art and $5
Algorithm 1: The EehEeh Sixteen promotions algorithm

1: procedure PReOMT\((T_{food}, T_{movies}, T_{art}, T_{books}, s_{\vec{A}_1}, s_{\vec{A}_2}, \ldots, s_{\vec{A}_N})\) 

2: \(P_{\text{food}} = \frac{T_{\text{food}}}{T_{\text{food}} + T_{\text{movies}} + T_{\text{art}} + T_{\text{books}}}\)

3: \(P_{\text{movies}} = \frac{T_{\text{movies}}}{T_{\text{food}} + T_{\text{movies}} + T_{\text{art}} + T_{\text{books}}}\)

4: \(P_{\text{art}} = \frac{T_{\text{art}}}{T_{\text{food}} + T_{\text{movies}} + T_{\text{art}} + T_{\text{books}}}\)

5: \(P_{\text{books}} = \frac{T_{\text{books}}}{T_{\text{food}} + T_{\text{movies}} + T_{\text{art}} + T_{\text{books}}}\)

6: Solve the system

\[0.4c_{\text{party}} + 0.7c_{\text{family}} + 0.2c_{\text{student}} + 0.05c_{\text{office}} = p_{\text{food}}\]

\[0.33c_{\text{party}} + 0.1c_{\text{family}} + 0.1c_{\text{student}} + 0.02c_{\text{office}} = p_{\text{movies}}\]

\[0.22c_{\text{party}} + 0.1c_{\text{family}} + 0.15c_{\text{student}} + 0.2c_{\text{office}} = p_{\text{art}}\]

\[0.05c_{\text{party}} + 0.1c_{\text{family}} + 0.55c_{\text{student}} + 0.73c_{\text{office}} = p_{\text{books}}\]

7: Assign \(\vec{x}_c = \begin{bmatrix} c_{\text{party}} \\ c_{\text{family}} \\ c_{\text{student}} \\ c_{\text{office}} \end{bmatrix}\)

8: Pick promotion \(A\) such that \(\langle \vec{x}_c, \vec{s}_A \rangle\) is highest.

9: Print promotion \(A\)

10: end procedure

on books. With promotions \(A_1, A_2, A_3\) and \(A_4\) targeted at customers with preferences \(s_{\vec{A}_1} = \begin{bmatrix} 1 \\ -3/4 \\ -1/4 \\ 0 \end{bmatrix}\), \(s_{\vec{A}_2} = \begin{bmatrix} 2/3 \\ -2/3 \\ 1/3 \\ 1/3 \end{bmatrix}\), \(s_{\vec{A}_3} = \begin{bmatrix} -1/2 \\ 1/2 \\ 1/2 \\ 0 \end{bmatrix}\) and \(s_{\vec{A}_4} = \begin{bmatrix} 0 \\ -1/2 \\ -1/2 \\ -1/2 \end{bmatrix}\).

**Answer:**

First, we normalize to get: \(p_{\text{food}} = 37.5\%, p_{\text{movies}} = 25\%, p_{\text{art}} = 6.25\%, p_{\text{books}} = 31.25\%\) which yields the following system of linear equations:

\[0.4c_{\text{party}} + 0.7c_{\text{family}} + 0.2c_{\text{student}} + 0.05c_{\text{office}} = 0.375\]

\[0.33c_{\text{party}} + 0.1c_{\text{family}} + 0.1c_{\text{student}} + 0.02c_{\text{office}} = 0.25\]

\[0.22c_{\text{party}} + 0.1c_{\text{family}} + 0.15c_{\text{student}} + 0.2c_{\text{office}} = 0.0625\]

\[0.05c_{\text{party}} + 0.1c_{\text{family}} + 0.55c_{\text{student}} + 0.73c_{\text{office}} = 0.3125\]

The solution to this system is \(\vec{x}_c = \begin{bmatrix} -0.02295082 \\ -0.22311475 \\ 3.18704918 \\ -1.94098361 \end{bmatrix}\).

Running inner products we get \(\langle \vec{x}_c, s_{\vec{A}_1} \rangle = -2.68704918033\), \(\langle \vec{x}_c, s_{\vec{A}_2} \rangle = 1.04278688525\), \(\langle \vec{x}_c, s_{\vec{A}_3} \rangle = 7.44366120219\) and \(\langle \vec{x}_c, s_{\vec{A}_4} \rangle = -1.08204918033\) and therefore, the promotion \(A_3\) will be printed.
(f) Will there ever be a customer for which the system devised in part [c] will yield no solutions or infinite solutions?

**Answer:** No. In Gaussian elimination, to have a system with no solutions or infinite solutions, there must be a row, in the augmented matrix that looks like \[
\begin{bmatrix}
0 & 0 & \cdots & 0 & | & a
\end{bmatrix}
\]
which means that if you run Gaussian elimination on the left hand side matrix representing the system in part [c]

\[
\begin{bmatrix}
0.4 & 0.7 & 0.2 & 0.05 \\
0.33 & 0.1 & 0.1 & 0.02 \\
0.22 & 0.1 & 0.15 & 0.2 \\
0.05 & 0.1 & 0.55 & 0.73
\end{bmatrix}
\]
you must get a row of zeros. But this is not the case here. This is good news, since we are guaranteed that our algorithm will not get stuck in ’an unexpected situation’ in step 6 (where we have to solve the system).

This fact was discussed in lecture in relation to linear dependence. This also means that the uniqueness of solutions to a system of linear equations is only a function of the left hand side (the experiment design), and not the right hand side (the measurements).