1. Mechanical Problems – Inverse

In each part, find the inverse of $A$ or determine that the inverse doesn’t exist.

(a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$
(b) $A = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix}$
(c) $A = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 0 & 4 \end{bmatrix}$
(d) $A = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$

2. Mechanical Problems – Eigenspaces

In each part, find the eigenspace of $M$ associated with the eigenvalue $\lambda$.

(a) $M = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$, $\lambda = 1$.
(b) $M = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$, $\lambda = 9$.
(c) $M = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$, $\lambda = 3$.

3. Eigenvalues and Special Matrices – Visualization

The following parts don’t require knowledge about how to find eigenvalues. Answer each part by reasoning about the matrix at hand.

(a) Does a rotation matrix in $\mathbb{R}^2$ have any eigenvalue $\lambda \in \mathbb{R}$?
(b) Does a reflection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?
(c) Does a projection matrix in $\mathbb{R}^2$ have any eigenvalues $\lambda \in \mathbb{R}$?
(d) If a matrix $M$ has an eigenvalue 0, what does this say about its null space? What does this say about the solutions of the system of linear equations $M\vec{x} = \vec{b}$?
4. Steady State Reservoir Levels

We have 3 reservoirs: \( A, B \) and \( C \). The pumps system between the reservoirs is depicted in Figure 1.

![Reservoir pumps system](image)

**Figure 1: Reservoir pumps system.**

(a) Write the transition matrix representing the pumps system in the problem.

(b) Assuming you start the pumps with the water levels of the reservoirs at \( A_0 = 129, B_0 = 109, C_0 = 0 \) (in kiloliters). What would be the steady state water levels (in kiloliters) according to the pumps system described in the problem?

*Hint:* If \( \vec{x}_{ss} = \begin{bmatrix} A_{ss} \\ B_{ss} \\ C_{ss} \end{bmatrix} \) is a vector describing the steady state levels of water in the reservoirs (in kiloliters), what happens if you fill the reservoirs \( A, B \) and \( C \) with \( A_{ss}, B_{ss} \) and \( C_{ss} \) kiloliters of water, respectively, and apply the pumps once?

*Hint II:* Note that the pumps system preserves the total amount of water in the reservoirs. That is, no water is lost or gained by applying the pumps.