1. **Gram-Schmidt and QR Factorization**

Compute the QR factorization of the following matrix:

\[
A = \begin{bmatrix}
1 & 1 & 0 \\
1 & 0 & 1 \\
0 & 0 & 1 \\
\end{bmatrix}
\]
2. QR Proofs

(a) Let $A = QR$, where $Q$ is an orthonormal matrix and $R$ is an upper triangular matrix. Show that if $A$ is invertible, then $R$ is invertible.

(b) Let $A = QR$, where $A$ has linearly independent columns, $Q$ is an orthonormal matrix, and $R$ is an upper triangular matrix. Show that $A$ and $Q$ have the same column space.

3. QR Factorization (Fall 2016 Final)

Recall that the solution to a linear least squares problem is a minimization of $\| \vec{b} - A\vec{x} \|^2$. Show that the approximation of $\vec{x}$, $\vec{x}$, in this linear least squares formula has an equivalent representation using the QR factorization of $A$, $(A = QR)$. In other words, express $\vec{x}$ in terms of $R$, $Q$, and $b$. Assume the matrix $A$ is full rank.