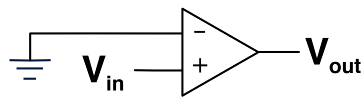
Solutions: Courtesy of John Noonan.

For each problem, assume the op-amp has a nominal gain of $G = 100$.

1. Op-Amps Without Feedback



(a) For the circuit above, write $H(\omega) = V_{out}/V_{in}$.

Solutions:

$V_{out} = G(V^+ - V^-)$

$V^- = 0$ $V^+ = V_{in}$

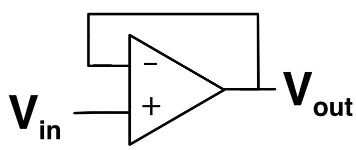
Thus, $V_{out} = G(V_{in} - 0) = GV_{in}$

$H(\omega) = \frac{V_{out}}{V_{in}} = G$

(b) If G is 20% lower that its nominal value, what is the percent error in $H(\omega)$ relative to nominal?

Solutions: Since $H(\omega) = G$, the percent error will be 20%.

2. Op-Amps With Feedback



(a) For the circuit above, approximate V_{out}/V_{in} using the op-amp golden rules.

Solutions:

Using the Golden Rules, we know that $V^+ = V^-$. And since V^- is connected to V_{out} in negative feedback, $V_{in} = V_{out}$. Thus, $\frac{V_{out}}{V_{in}} = 1$.

(b) Now write V_{out}/V_{in} **without** the second op-amp golden rule (you can still assume no current flows into the amplifier inputs). How close is the result to the approximation from (a)? (Give a percentage.)

Solutions:

$V_{out} = G(V_{in} - V_{out})$

$V_{out} + GV_{out} = GV_{in}$

$(1+G)V_{out} = GV_{in}$

$\frac{V_{out}}{V_{in}} = \frac{G}{1+G}$

Thus, as G approaches ∞, $\frac{V_{out}}{V_{in}}$ approaches 1, in which case $V_{out} = V_{in}$.

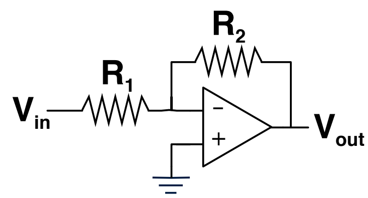
Percent error in $\frac{V_{out}}{V_{in}}$ relative to the approximation from (a): $(1 - \frac{G}{1+G}) * 100\% = (1 - \frac{100}{101}) * 100\% = 1.0\%$.

(c) Now assume G is 20% lower that its nominal value. What is the percent error in V_{out}/V_{in} relative to the approximation from (a)?

Solutions:

Percent error in $\frac{V_{out}}{V_{in}}$ relative to the approximation from (a): $(1-\frac{0.8G}{1+0.8G})*100\% = (1-\frac{80}{81})*100\% = 1.23\%$.

3. Inverting Amplifier



(a) For the circuit above, approximate V_{out}/V_{in} using the op-amp golden rules.

Solutions: Using the golden rules, we get that $\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$.

(b) Now write V_{out}/V_{in} without the second op-amp golden rule (you can still assume no current flows into the amplifier inputs).

Solutions:

$\frac{V_{in}-V_A}{R_1} = \frac{V_A-V_{out}}{R_2}$

Also, $V_{out} = G(0-V_A)$

$V_{out} = -GV_A$

$V_A = -\frac{V_{out}}{G}$

$\frac{1}{R_1}V_{in} - \frac{1}{R_1}*-\frac{V_{out}}{G} = -\frac{1}{R_2}\frac{V_{out}}{G} - \frac{V_{out}}{R_2}$

$\frac{1}{R_1}V_{in} = -(\frac{1}{R_1}\frac{1}{G}+\frac{1}{R_2}\frac{1}{G}+\frac{1}{R_2})V_{out}$

$\frac{V_{out}}{V_{in}} = \frac{1}{R_1}\frac{-1}{\frac{1}{R_1}\frac{1}{G}+\frac{1}{R_2}\frac{1}{G}+\frac{1}{R_2}}$

$= \frac{-1}{\frac{1}{G}+\frac{R_1}{R_2}\frac{1}{G}+\frac{R_1}{R_2}}$

(c) If $R_1 = 100\Omega$ and $R_2 = 500\Omega$, find the gain of the circuit using the models from both (a) and (b). What is the percent error?

Solutions:

a: $-\frac{R_2}{R_1} = -\frac{500}{100} = -5$

b: $\frac{-1}{\frac{1}{100}+\frac{1}{5}\frac{1}{100}+\frac{100}{500}} = -4.72$

About 5.6% error.

(d) Now assume G is 20% lower that its nominal value and recalculate the gain. What is the new percent error compared to (a)?

Solutions:

a: -5

b: $\frac{-1}{\frac{1}{80}+\frac{1}{5}\frac{1}{80}+\frac{100}{500}} = -4.65$

About 7.0% error.