9/2 Discussion

- Introduce
  - Name, email, sections, off hrs
  - 'Wait' policy

- Outline for today
  - Linear Alg & DFT practice/review
  - DFT visualization
  - Time & Frequency

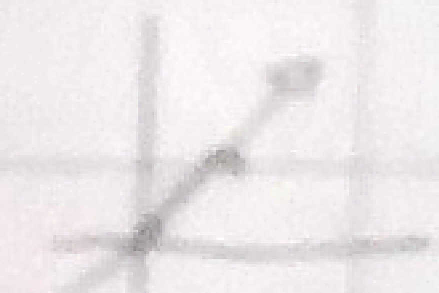
- First any pressing questions from lecture?

Linear Alg

Easy question: what is a dot product?

- Dot product is a measure of how much one vector is like another. Don't not comfortable w/ this concept

For example,



$$\langle 1,1,0\rangle \cdot \langle 2,2,0\rangle = [1\ 1\ 0]\begin{bmatrix}2\\2\\0\end{bmatrix} = 2\cdot1 + 2\cdot1 + 0 = 4$$

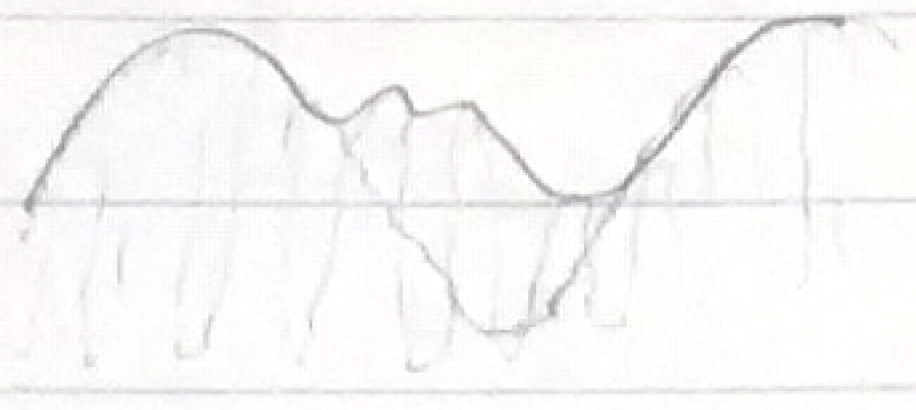
but, if magnitude is same but phase or angle is diff.

$$\langle 1,1,0\rangle \cdot \langle 0,2,2\rangle = [1\ 1\ 0]\begin{bmatrix}0\\2\\2\end{bmatrix} = 2 \leftarrow \text{less similar}$$

- and if I wanted to make the math harder then I would normalize the vectors & a dot product of 1 would be most similar

Hard question: What is a DFT?

- A DFT is how much a signal looks like different frequencies



Since we can recreate any function out of sin & cos functions

- A dot product is a very good measure of similarness, so we use it in DFT.

<u>Ex</u> Practice w/ complex dot products

Say we have a small set of 4 periodic samples

$P = 4$

Then there are 4 frequencies that can be represented in those samples

$\omega = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

Each of those sinusoids can be represented in 4 points as

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ j \\ -1 \\ -j \end{bmatrix} \quad \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix}$$

Say the sample is $[1 \ 0 \ 0 \ 0]$ because easy math is the best.

We will take the DFT by performing the dot product w/ each sinusoid

OFFICIAL MATH way of writing

$$x \circ y = \sum_{n=0}^{N-1} x(n) \cdot y(n)^{*}$$ ← please note the star

star = complex conj.

$$= \begin{bmatrix} x(0) & x(1) & \cdots & x(N-1) \end{bmatrix} \begin{bmatrix} y(0)^{*} \\ \vdots \\ y(N-1)^{*} \end{bmatrix}$$

So, Easy one first

$$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -j \\ -1 \\ +j \end{bmatrix} = 1$$

Note negative

$$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = 1$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ +j \\ -1 \\ -j \end{bmatrix} = 1$$

Also, fun experiment

$$\begin{bmatrix} 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = 1 + j - 1 - j = 0$$

$$\begin{bmatrix} 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -j \\ -1 \\ +j \end{bmatrix} = 1 + 1 + 1 + 1 = 4$$

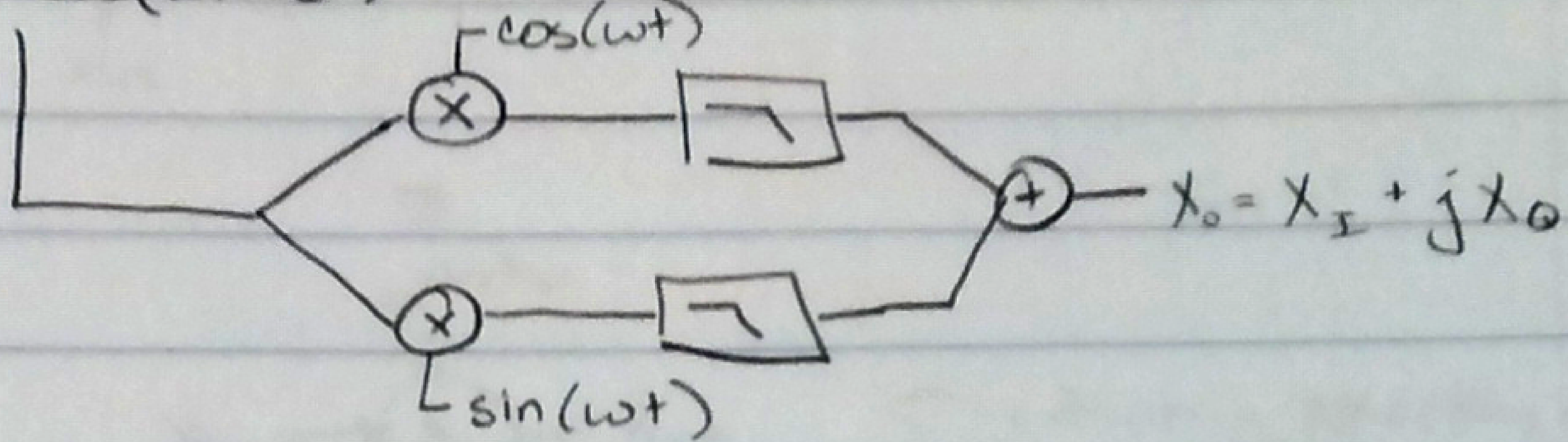
$$\begin{bmatrix} 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = 1 - j - 1 + j = 0$$

$$\begin{bmatrix} 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ +j \\ -1 \\ -j \end{bmatrix} = 1 - 1 + 1 - 1 = 0$$

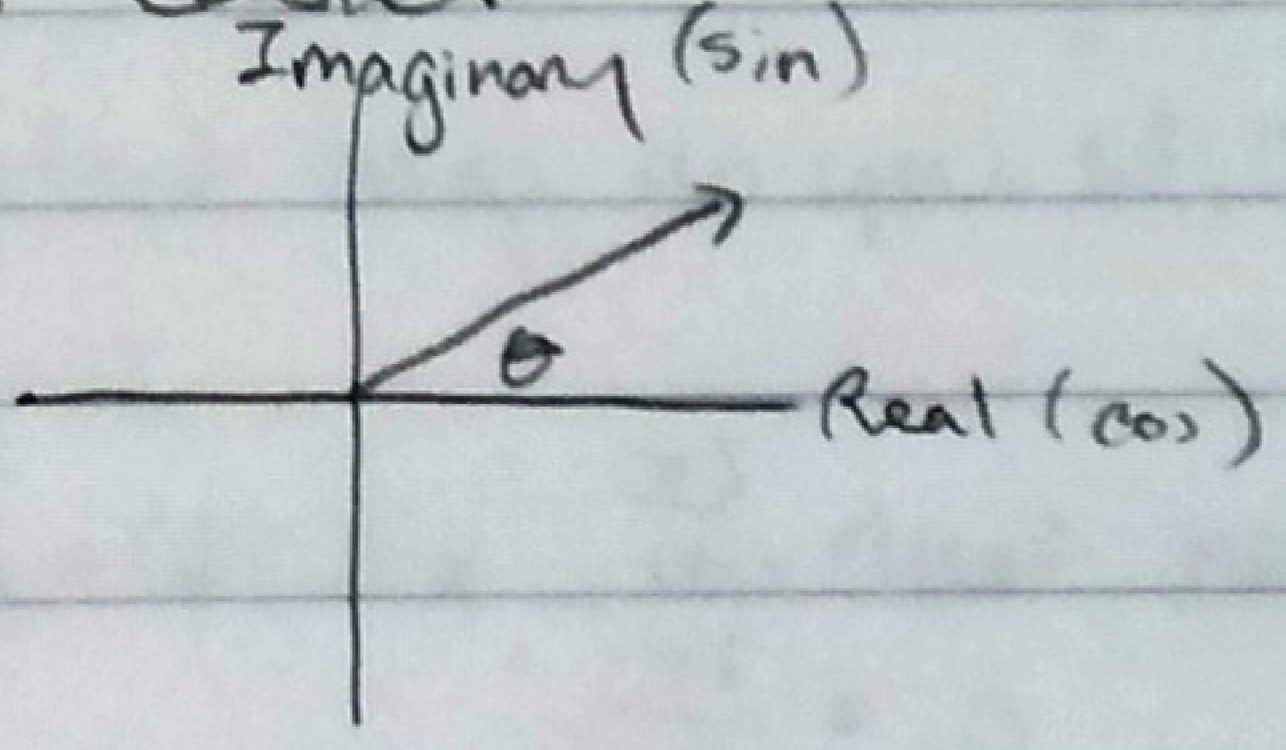
Complex Dot products are easy! Right? Questions?

Lets look a little closer at this whole sin, cos thing, & I am going to try to relate it to the software defined radios you guys did last semester.

$X_R = X_a \cdot \cos(\omega t + \theta)$

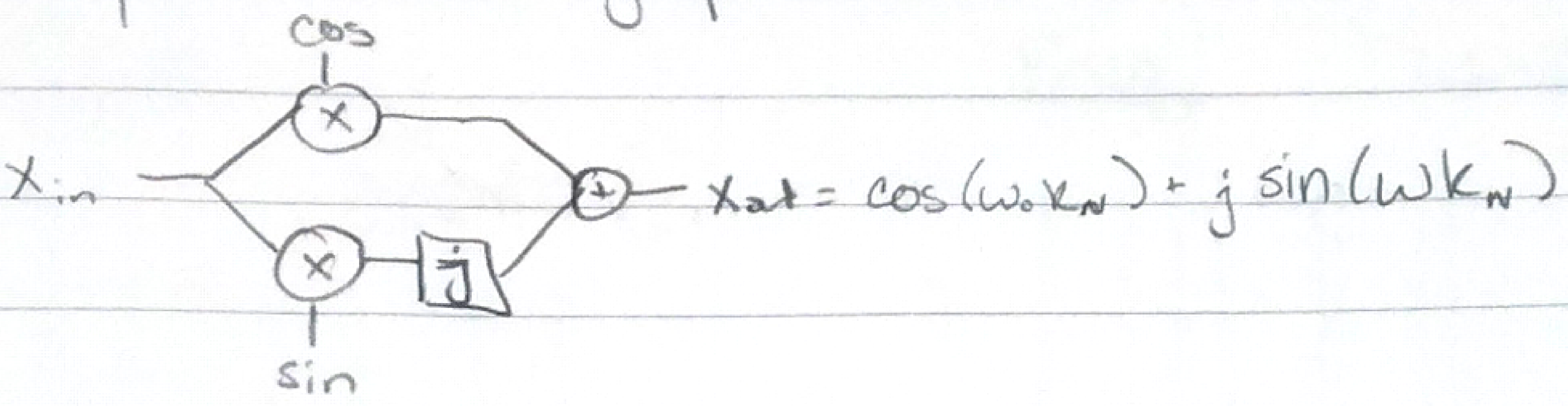


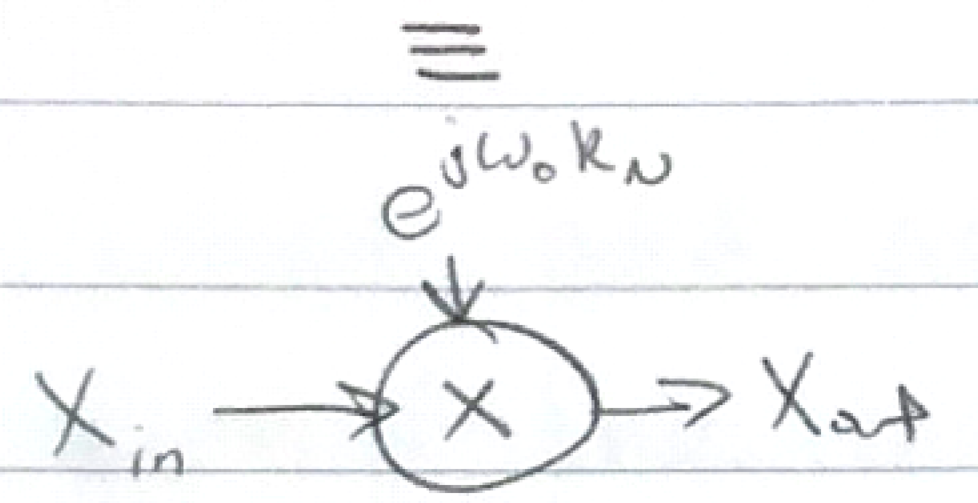
This should look familiar, but lets take a moment to think intuitively about what is going on.
By multiplying by sin & cos, we can break the X_R equation into a real & imaginary part.
Why do we use imaginary parts? PAUSE To be honest, it is mostly just because it makes thinking about phase easier



If θ is 0° then the number will be real, if θ is 90° then the number is completely imaginary. This also means we can think of them as the perpendicular parts of sin & cos, (Draw sin & cos)

So, it should feel very comfortable when I say that that graph looks a lot like a DFT.





← which is exactly a DFT

Does this make sense?

Finally, lets look at sampling & window size, & the relation between time & frequency from that.

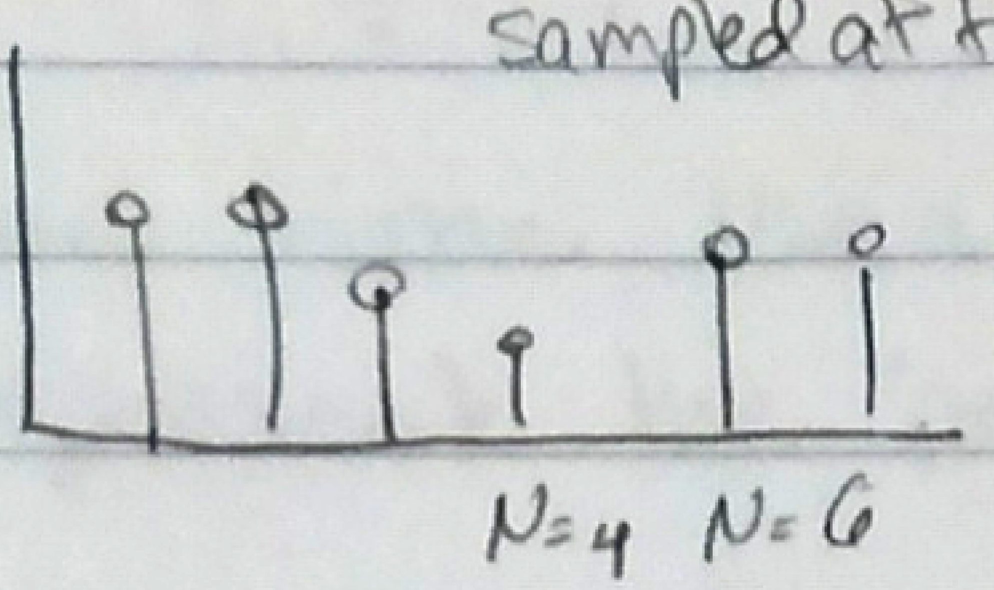
Lets go back to that 4 sample window $N=4$.
What very important thing do you still need?
sampling time
lets say $F_s = 4\text{ Hz}$, so that my window is 1s wide.

Those ω frequencies that fit into my window
$\omega = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
without even converting them to continuous time, we can see that they are pretty wide frequency bins

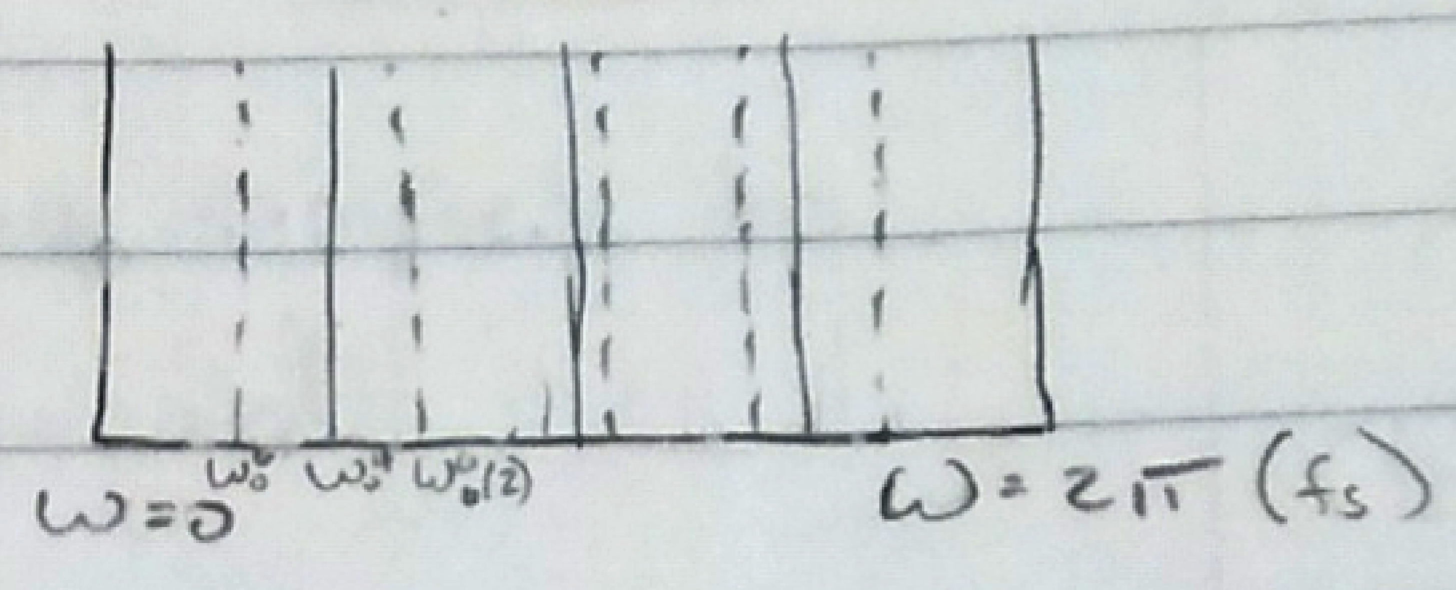
If we were to make the window larger

Math shows clearly $\omega_0 = 2\pi/N$

Time



Freq



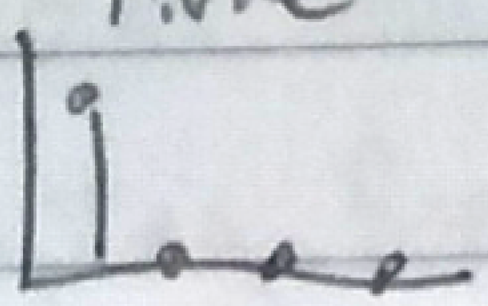
Then we would add another frequency bin & make each of the bins more narrow.

Wider time window ⇒ more freq resolution

Now time & freq have this really cool duality thing that they do. Lets table this thought for just a moment so that we can look back at our example w/ the dot products.

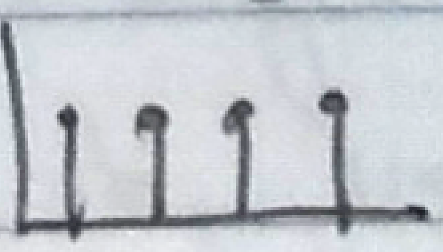
Notice, that when we had a super "narrow" signal (this particular one is a delta function) the frequency graph shows a value in every bin

Time

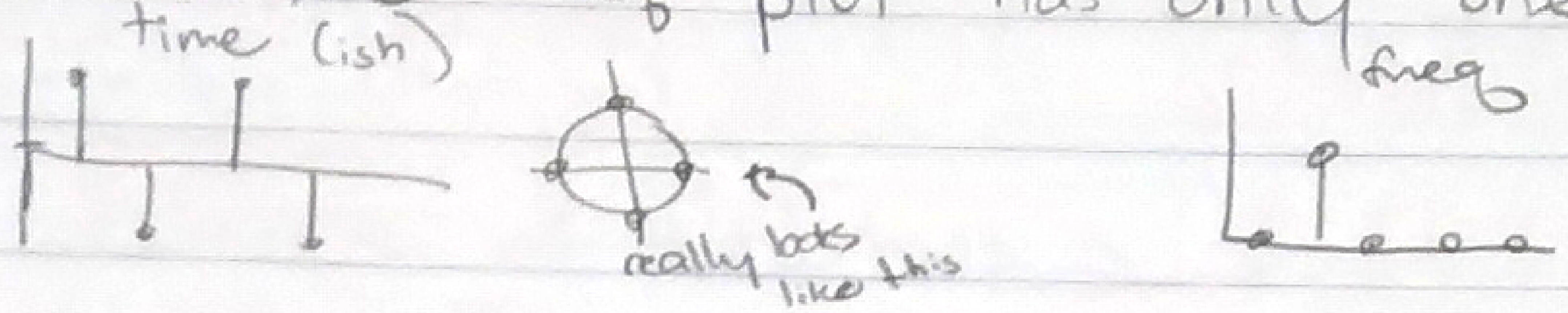


⇒

Freq

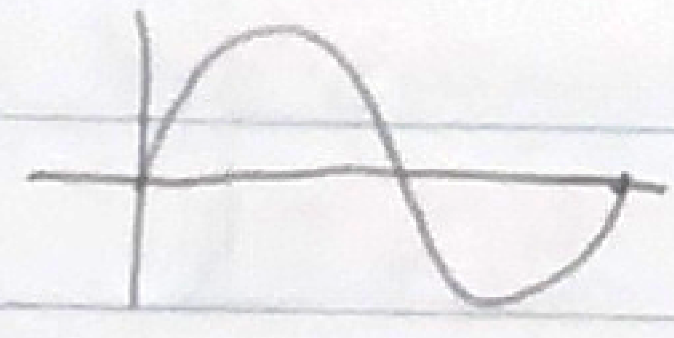


But, when we choose an exact frequency so that the freq plot has only one point

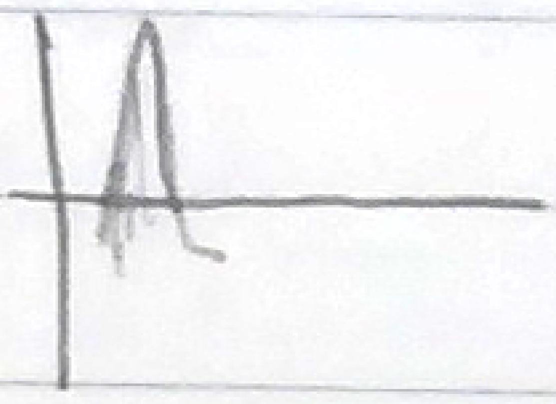


The time uses all of the samples to represent the 'perfect' sinusoid

This is to give you sort of an intuition about how narrow things in time, take a lot of room in freq to represent. But narrow things in freq take a lot of time samples to represent.

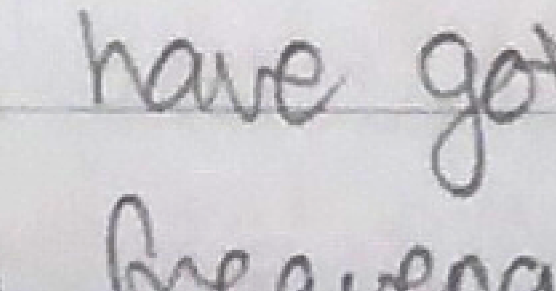


vs



Now, to put our brain back a couple steps. We just went over how a wider window gives more detail in frequency bins.

However, that window doesn't mean much yet. I could have gotten a larger N by increasing my sampling frequency, or by just adding a zero at the end.



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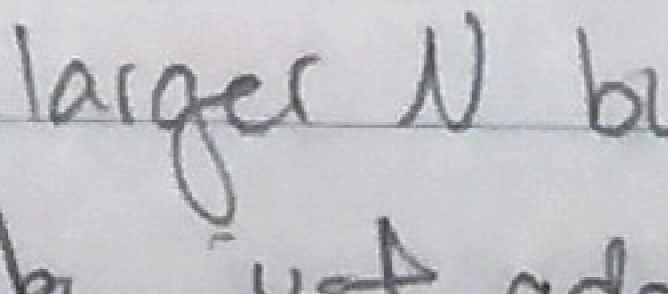
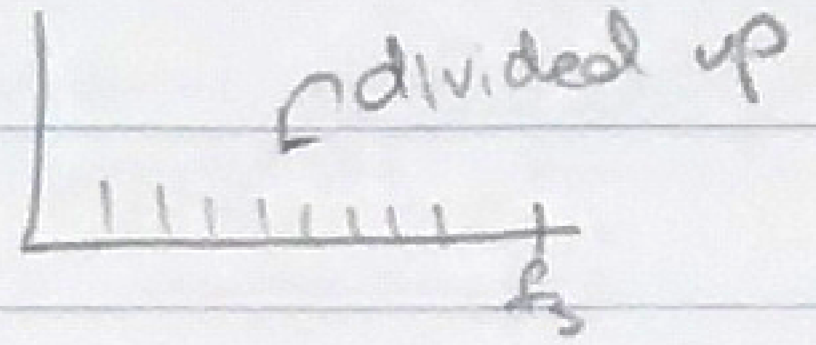


So, lets see what happens when I increase the sampling frequency.

(width)
The resolution of the bins is:

$$\Delta f = \frac{F_s}{N}$$



Where do I get that? Math wise

$$N = T f_s$$

The number of samples I get is just the frequency of collection times the amount of time I collect for

so,

$$T = \frac{N}{f_s}$$

and the resolution of the bin

$$\Delta f = \frac{1}{T} = \frac{F_s}{N}$$

Other way of thinking about that is that my graph will show all the way up to my f_s so divide by N

So, if I increase my sampling freq, what happens to the bins?

Frequency bin size increases

What happens if I increase my window without changing the sampling frequency?

The bin size goes down