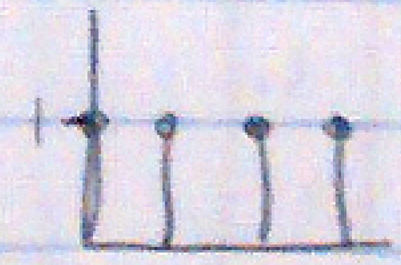
Discussion 2

Questions about lecture? How are you feeling about freq domain, DFT, & such?

Visualizing b/w time & freq:

Last time, I mentioned the duality b/w time & freq, where wide things in time are narrow in freq & vise versa.

We did a 4 point DFT & got a graph like this



What was this in time? a delta function, yes.

But lets work through the math real quick just to prove it to ourselves

What is the IDFT eqn?

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_k \, \Psi_k(n)$$

Last time I gave you the vectors for $\varphi_k(n)$ & pushed off the explanation for later in lecture. Well, now, you should know how to find each $\varphi_k(n)$ can someone give them to me?

$$\begin{bmatrix} e^{j\omega_0(0)(0)} \\ e^{j\omega_0(0)(1)} \\ \vdots \end{bmatrix} \quad \begin{bmatrix} \ddots \end{bmatrix} \quad \cdots$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ j \\ -1 \\ -j \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 \\ -j \\ -1 \\ j \end{bmatrix}$$

Excellent!

Now lets plug these in

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j\omega_0 nk}$$

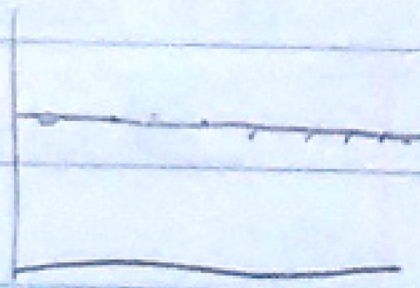
$$x(0) = \tfrac{1}{4}(1\cdot 1 + 1\cdot 1 + 1\cdot 1 + 1\cdot 1)$$
$$x(1) = \tfrac{1}{4}(1\cdot 1 + 1\cdot j + 1\cdot -1 + 1\cdot -j)$$
$$x(2) = \tfrac{1}{4}(\quad \vdots$$

Sure enough, we get a $\delta(n)$!

$x(n) =$ 

Now, lets play around w/ this a little in ipython

I have constructed an X_k in ipython (this will be in the discussion notes if you want to run it yourself)

$X_k = [1\ 1 \cdots\cdots 1\ 1]$ 

Took the IDFT

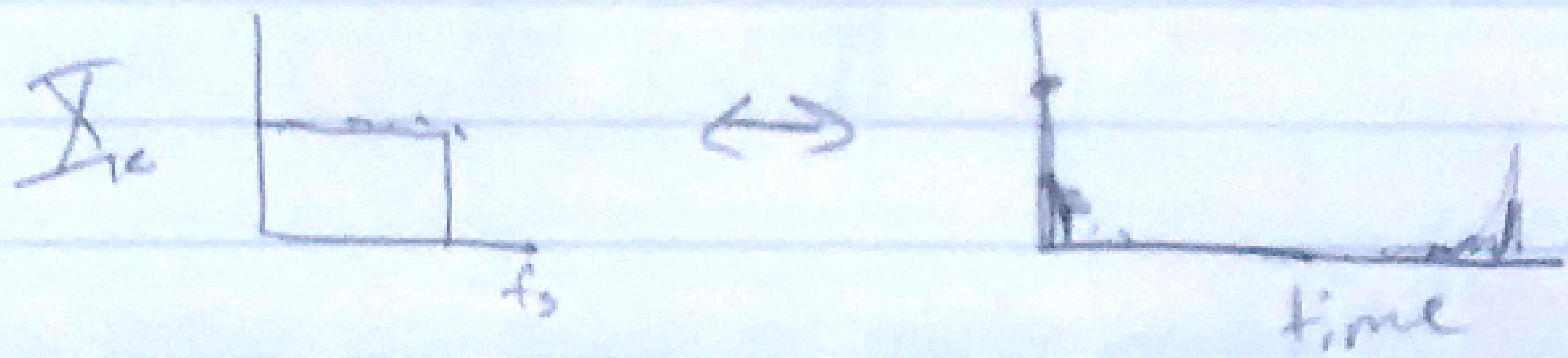
$x = np.fft.ifft(X)$

And sure enough got a δ function. Instead of looking at it, lets listen to it.

Note that there is 1 click & then nothing.

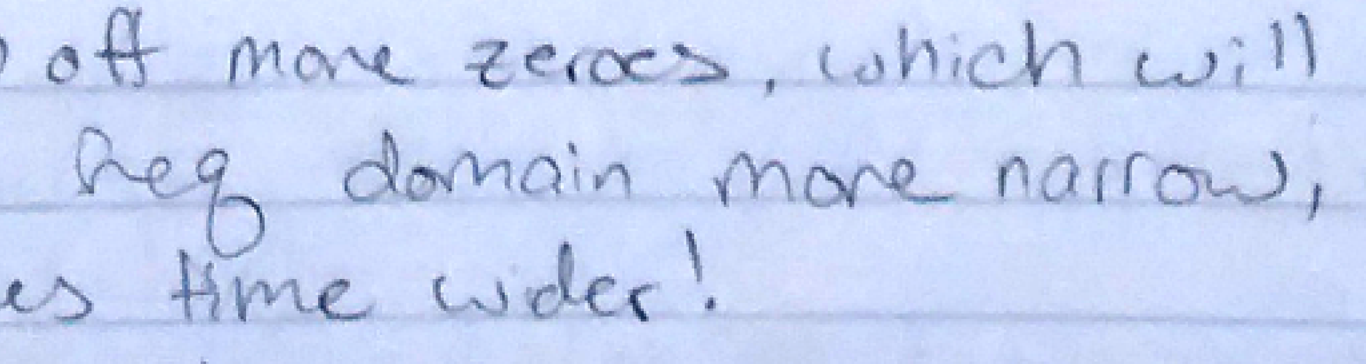
Now, lets do a frequency domain filter.
What does that look like? If I
give you X_k what do you do to it
to cut out some high freq?

Zero out some freq components. That
will remove all of that freq from the
time domain



Now listen. It's a little hard to
hear, but there is some crackle
at the end.

How do we make it more obv?
Chop off more zeroes, which will make
the freq domain more narrow, which
makes time wider!

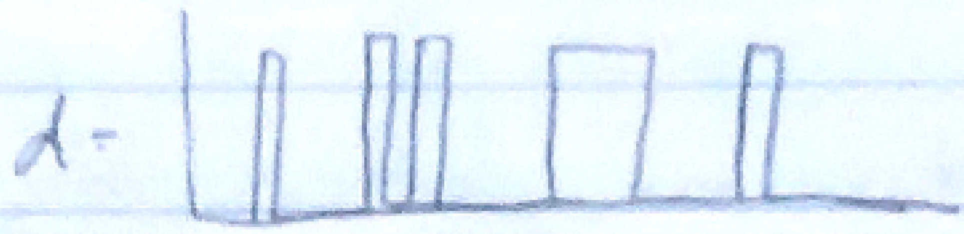


Now you can hear two clicks

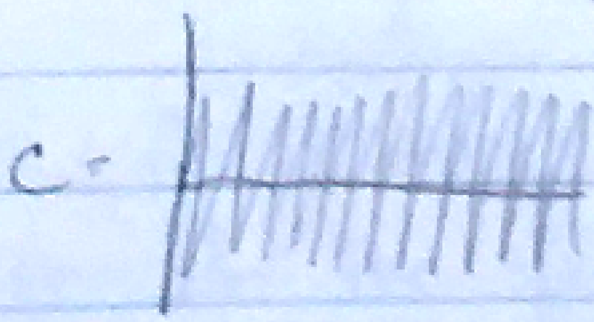
Now, lets move away from audio & look at wireless signals.

Does anyone remember On-off Keyed wireless system?

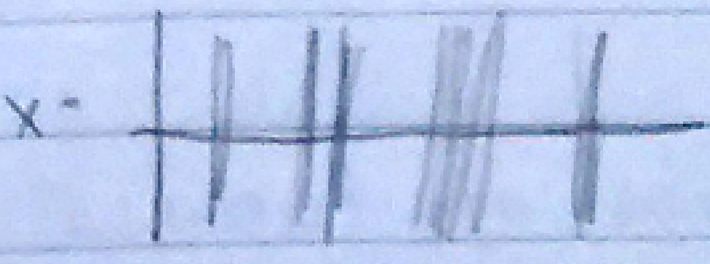
I take some data, that I want to send at, say, 1 MHz



Which is neither fast enough nor regular enough to send on it's own, so I mix it w/ a pure freq that is better for sending w/, this is my carrier freq.

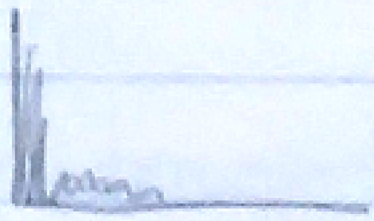


I multiply them & get



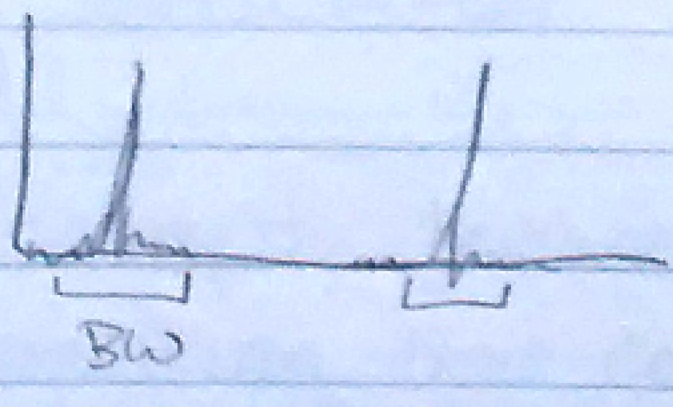
Now, what does the dft of each of these look like? C = pure tone

D I will give you because it is hard



So together they are what?
Well a multiplication in time is what in freq? convolution

X =

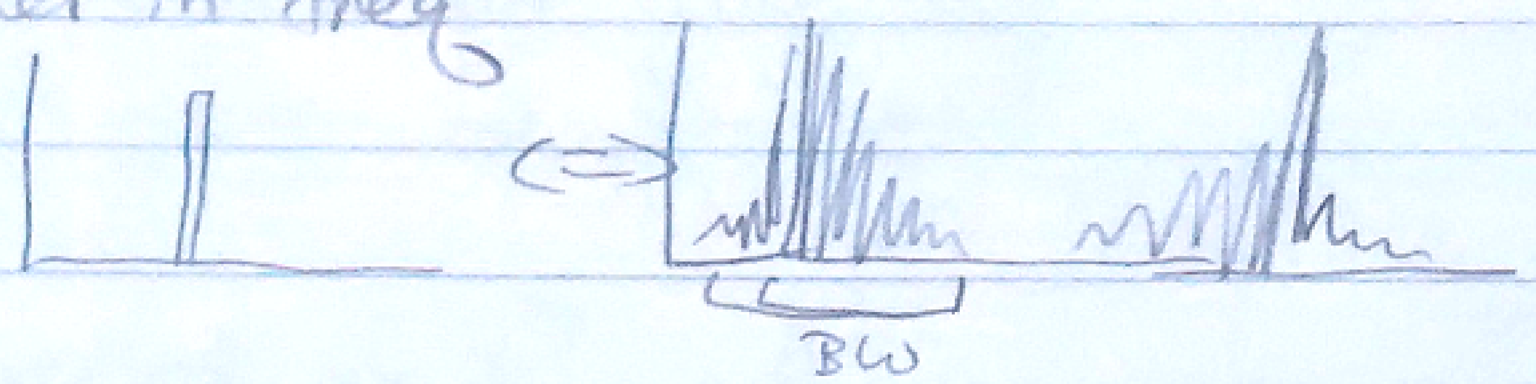


The width of the spike is the Bandwidth, the width of the frequency band that I take up in order to send my signal accurately.

Now, say I want to send my data faster, at 10MHz instead of 1

What do we expect will happen?

It is more narrow in time so it will be wider in freq



You have been taught before that in order to send data faster, you need more BW, but now we can look at the frequency domain to see just how much BW we are taking up by sending data faster.

Stuff like this is impt, because if you want to sell something that communicates wirelessly, you have to follow rules & keep within your assigned BW & not scream in other people's BW.

Now, let's move away from time/freq duality & talk about window size.

In class, you should have gone over some $N=8$ DFT for $x(n) = e^{j\frac{\pi}{2}n}$

What is the period of x? 4

So a DFT of length 8 works great. As you saw in class, taking the DFT gives you one spike in freq domain & then perfectly reconstructs the periodic signal.

What if we take a DFT for $N=6$? Predictions?

$$x = [1 \;\; j \;\; -1 \;\; -j \;\; 1 \;\; j]$$

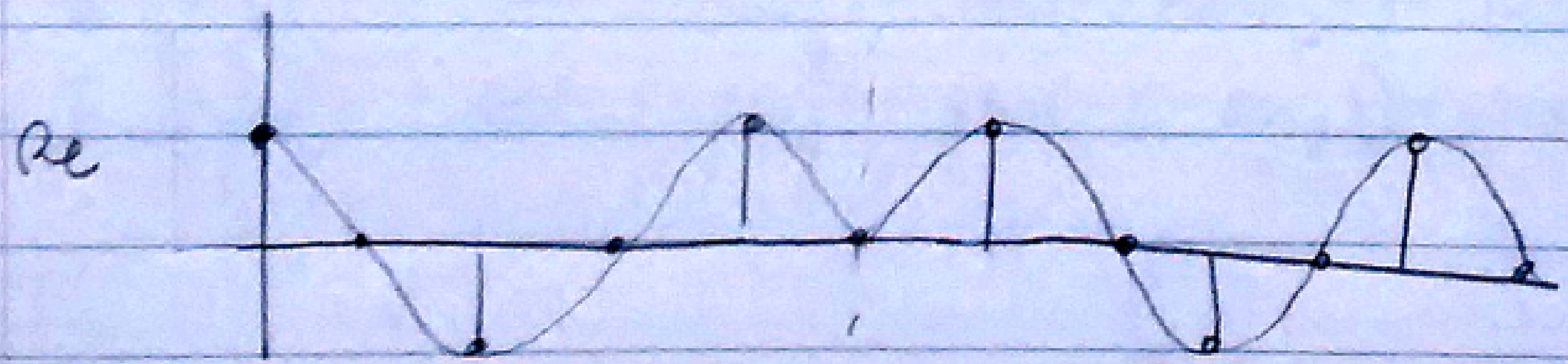
$$\psi = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{1}{2}+j\frac{\sqrt{3}}{2} & -\frac{1}{2}+j\frac{\sqrt{3}}{2} & -1 & -\frac{1}{2}-j\frac{\sqrt{3}}{2} & \frac{1}{2}-j\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2}+j\frac{\sqrt{3}}{2} & -\frac{1}{2}-j\frac{\sqrt{3}}{2} & 1 & -\frac{1}{2}+j\frac{\sqrt{3}}{2} & -\frac{1}{2}-j\frac{\sqrt{3}}{2} \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\frac{1}{2}-j\frac{\sqrt{3}}{2} & -\frac{1}{2}+j\frac{\sqrt{3}}{2} & 1 & -\frac{1}{2}-j\frac{\sqrt{3}}{2} & -\frac{1}{2}+j\frac{\sqrt{3}}{2} \\ 1 & \frac{1}{2}-j\frac{\sqrt{3}}{2} & -\frac{1}{2}-j\frac{\sqrt{3}}{2} & -1 & -\frac{1}{2}+j\frac{\sqrt{3}}{2} & \frac{1}{2}+j\frac{\sqrt{3}}{2} \end{bmatrix}$$

$$X_k = \begin{bmatrix} .408 + .408j \\ .408 + .109j \\ .408 - .109j \\ .408 - .408j \\ .408 - 1.5j \\ .408 + 1.5j \end{bmatrix}$$

Reconstruction will be perfect, b/c DFT is reversible

$x(n) = [\,1 \quad j \quad -1 \quad -j \quad 1 \quad j\,]$

However, if we try to make it periodic again,



DFT perfectly reconstructs whatever it is given, but if you chop the window so that your signal is no longer periodic, then it will be harder to represent cleanly in freq domain & is not periodic in time.