Bode Plot Examples

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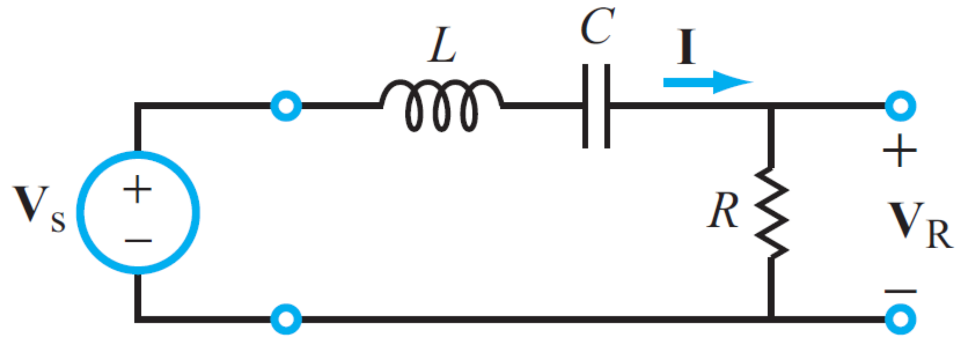
Bandpass RLC Filter revisited

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j(\omega L - \frac{1}{\omega C})} = \frac{j\omega C\mathbf{V}_s}{(1-\omega^2 LC) + j\omega RC}$$



$$\mathbf{H}_{BP}(\omega) = \frac{\mathbf{V}_R}{\mathbf{V}_s} = \frac{R\mathbf{I}}{\mathbf{V}_s} = \frac{j\omega RC}{(1-\omega^2 LC) + j\omega RC}$$

$$M_{BP}(\omega) = |\mathbf{H}_{BP}(\omega)| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}}$$

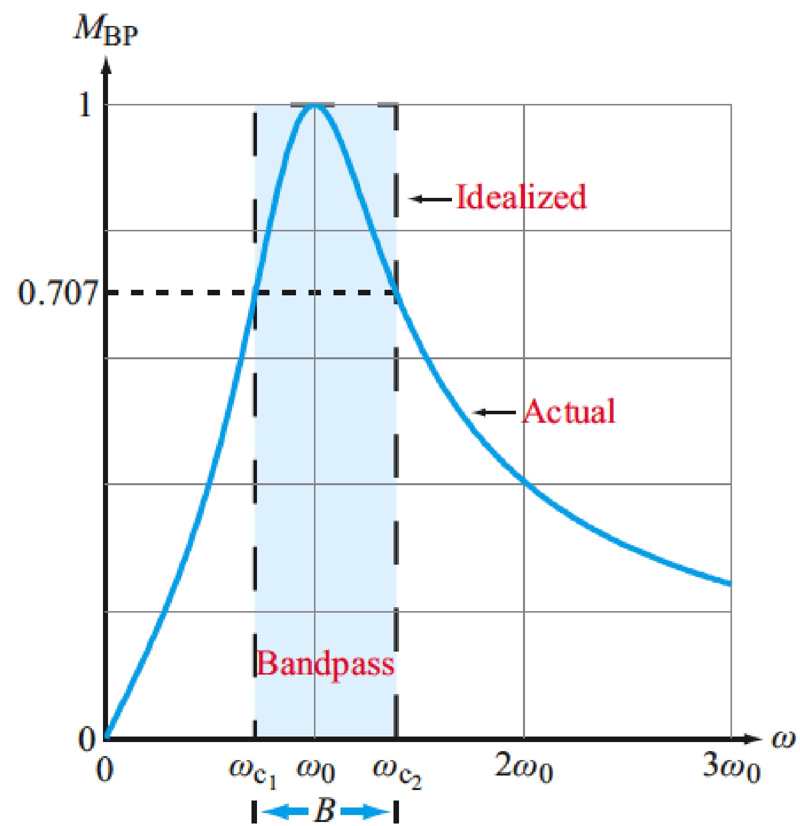
$$\phi_R(\omega) = 90^\circ - \tan^{-1}\left[\frac{\omega RC}{1-\omega^2 LC}\right]$$

$$\omega_{c_1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}},$$

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$

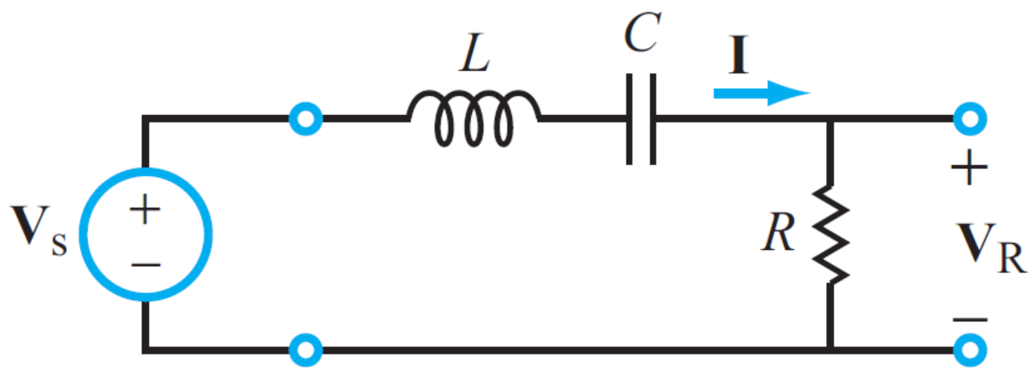
$$\omega_{c_2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}.$$

$$B = \omega_{c_2} - \omega_{c_1} = \frac{R}{L}.$$



Bandpass RLC Filter (cont.)

$$Q = 2\pi \left(\frac{W_{\text{stor}}}{W_{\text{diss}}} \right) \bigg|_{\omega=\omega_0}$$



$$v_s(t) = V_0 \cos \omega_0 t \quad \longleftrightarrow \quad \mathbf{V}_s = V_0.$$

- W_{stor} is *the maximum energy that can be **stored*** in the circuit at resonance ($\omega = \omega_0$)
- W_{diss} is *the **energy dissipated*** by the circuit during a single period *T.*

$$\mathbf{Z} = R + j\omega_0 L - j/\omega_0 C$$
$$= R \qquad (@\ \omega_0 = 1/\sqrt{LC})$$

and

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{\mathbf{V}_s}{R} = \frac{V_0}{R}.$$

The time-domain current then is given by

$$i(t) = \mathfrak{Re}\left[\frac{V_0}{R}\, e^{j\omega_0 t}\right] = \frac{V_0}{R} \cos \omega_0 t.$$

Bandpass RLC Filter (cont.)

Derivation of Q

$$i(t) = \Re\mathfrak{e}\left[\frac{V_0}{R}\, e^{j\omega_0 t}\right] = \frac{V_0}{R}\cos\omega_0 t$$

$$w_L(t) = \frac{1}{2}L\, i_L^2(t) = \frac{V_0^2 L}{2R^2}\cos^2\omega_0 t \qquad \text{(J)}$$

$$w_C(t) = \frac{1}{2}C\, v_C^2(t) = \frac{1}{2}C\left(\frac{1}{C}\int i\, dt\right)^2$$

$$= \frac{1}{2}C\left(\frac{V_0}{\omega_0 RC}\sin\omega_0 t\right)^2$$

$$= \frac{V_0^2 L}{2R^2}\sin^2\omega_0 t \qquad \text{(J)}.$$

$$W_{\text{stor}} = w_L(t) + w_C(t) = \frac{V_0^2 L}{2R^2}[\cos^2\omega_0 t + \sin^2\omega_0 t]$$

$$= \boxed{\frac{V_0^2 L}{2R^2}}. \qquad (9.59)$$

$$W_{\text{diss}} = \int_0^T p_R\, dt = \int_0^T i^2 R\, dt$$

$$= \int_0^{2\pi/\omega_0} \frac{V_0^2}{R}\cos^2\omega_0 t\, dt = \boxed{\frac{\pi V_0^2}{\omega_0 R}}$$

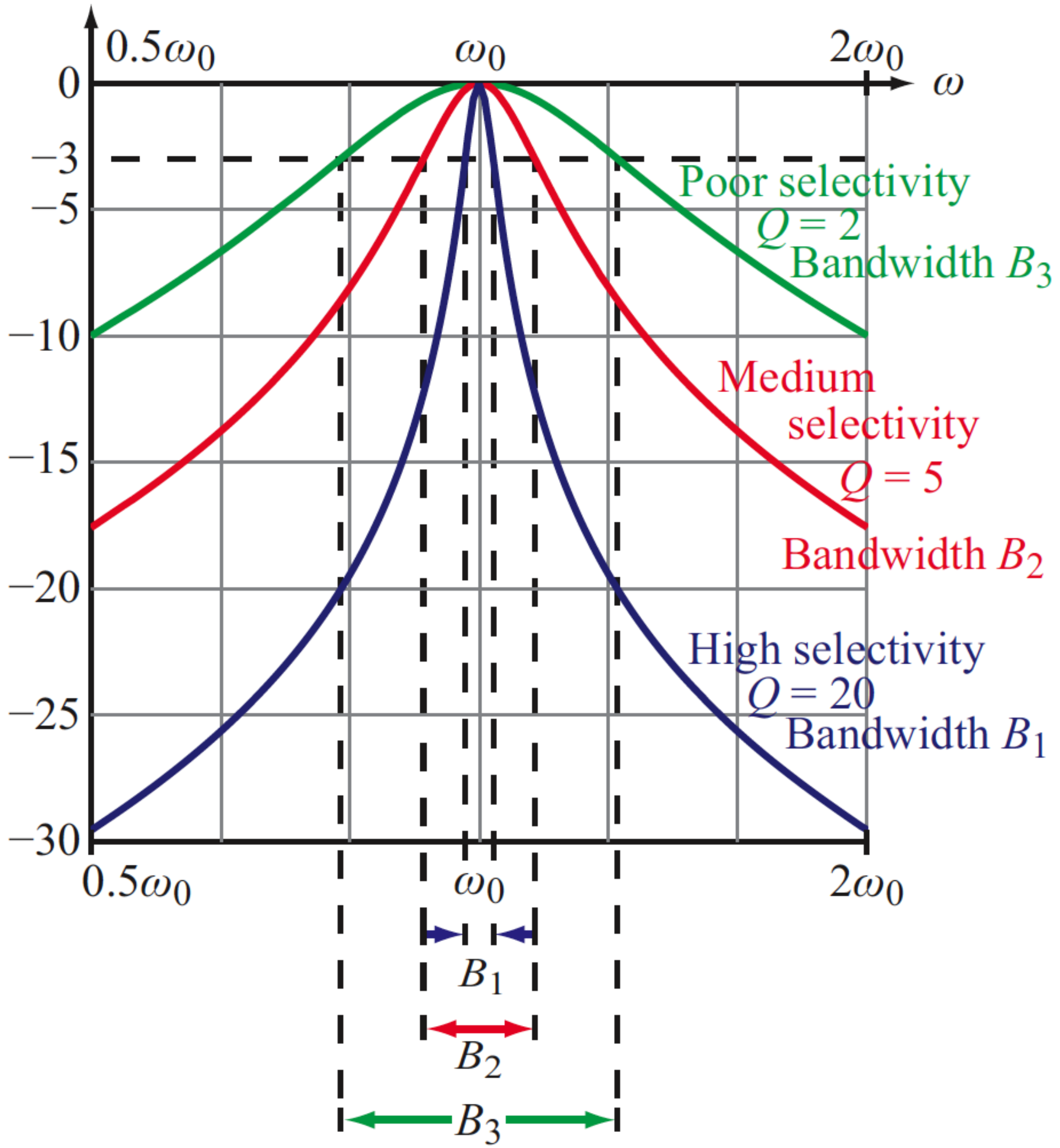
$$Q = 2\pi\left(\frac{W_{\text{stor}}}{W_{\text{diss}}}\right)\Bigg|_{\omega=\omega_0},$$

$$Q = \frac{\omega_0 L}{R} = \frac{\omega_0}{B},$$

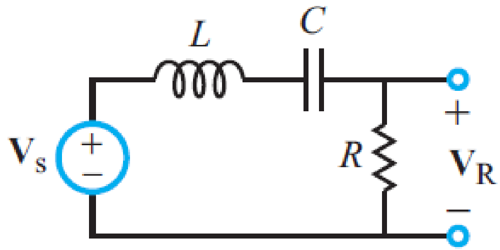
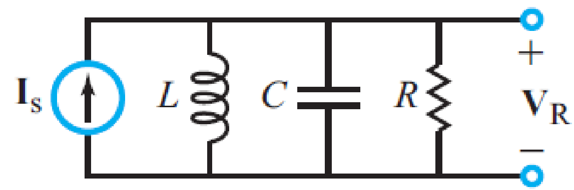
Resonant frequency

Bandwidth

Bandpass Filter



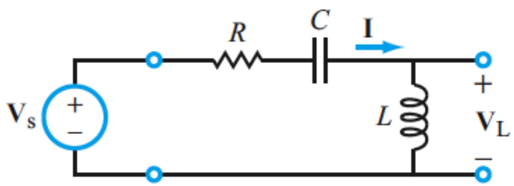
RLC Filters

RLC Circuit		
Transfer Function	$\mathbf{H} = \dfrac{\mathbf{V}_R}{\mathbf{V}_s}$	$\mathbf{H} = \dfrac{\mathbf{V}_R}{\mathbf{I}_s}$
Resonant Frequency, ω_0	$\dfrac{1}{\sqrt{LC}}$	$\dfrac{1}{\sqrt{LC}}$
Bandwidth, B	$\dfrac{R}{L}$	$\dfrac{1}{RC}$
Quality Factor, Q	$\dfrac{\omega_0}{B} = \dfrac{\omega_0 L}{R}$	$\dfrac{\omega_0}{B} = \dfrac{R}{\omega_0 L}$
Lower Half-Power Frequency, ω_{c_1}	$\left[-\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$	$\left[-\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$
Upper Half-Power Frequency, ω_{c_2}	$\left[\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$	$\left[\dfrac{1}{2Q} + \sqrt{1 + \dfrac{1}{4Q^2}}\right]\omega_0$

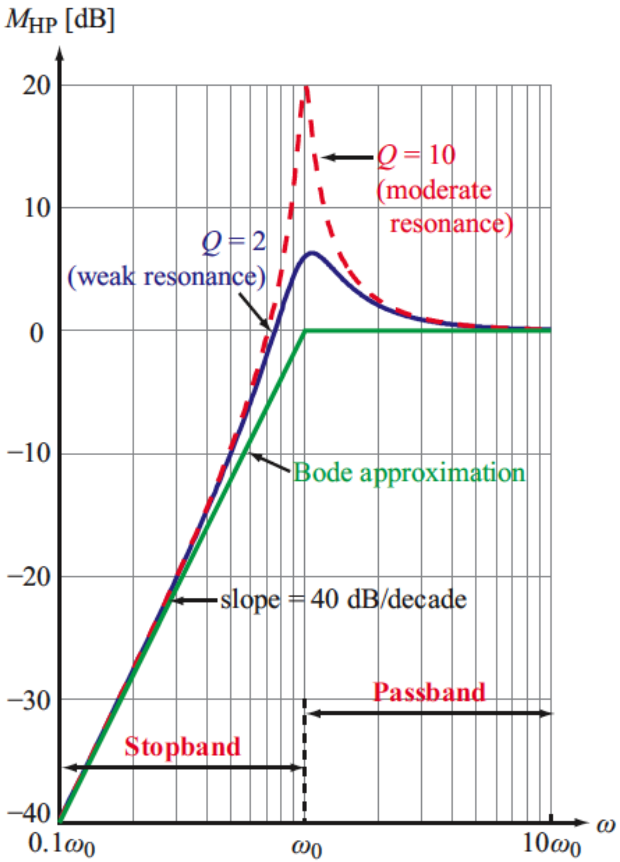
Notes: (1) The expression for Q of the series RLC circuit is the inverse of that for Q of the parallel circuit. (2) For $Q \geq 10$, $\omega_{c_1} \simeq \omega_0 - \frac{B}{2}$, and $\omega_{c_2} \simeq \omega_0 + \frac{B}{2}$.

RLC Filters, con't

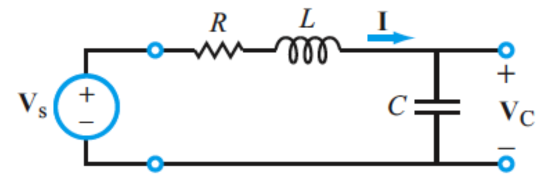
Highpass Filter



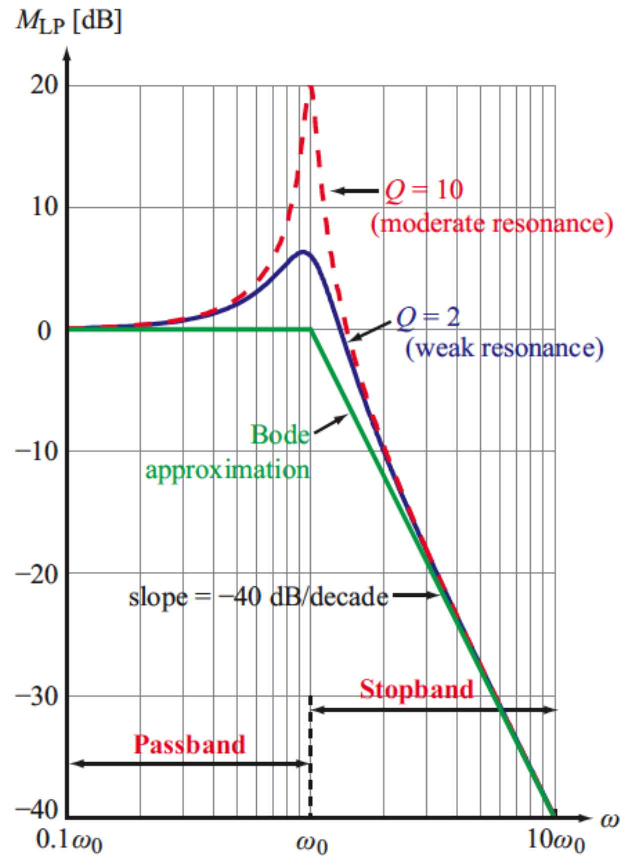
(a) $H_{HP} = V_L / V_s$



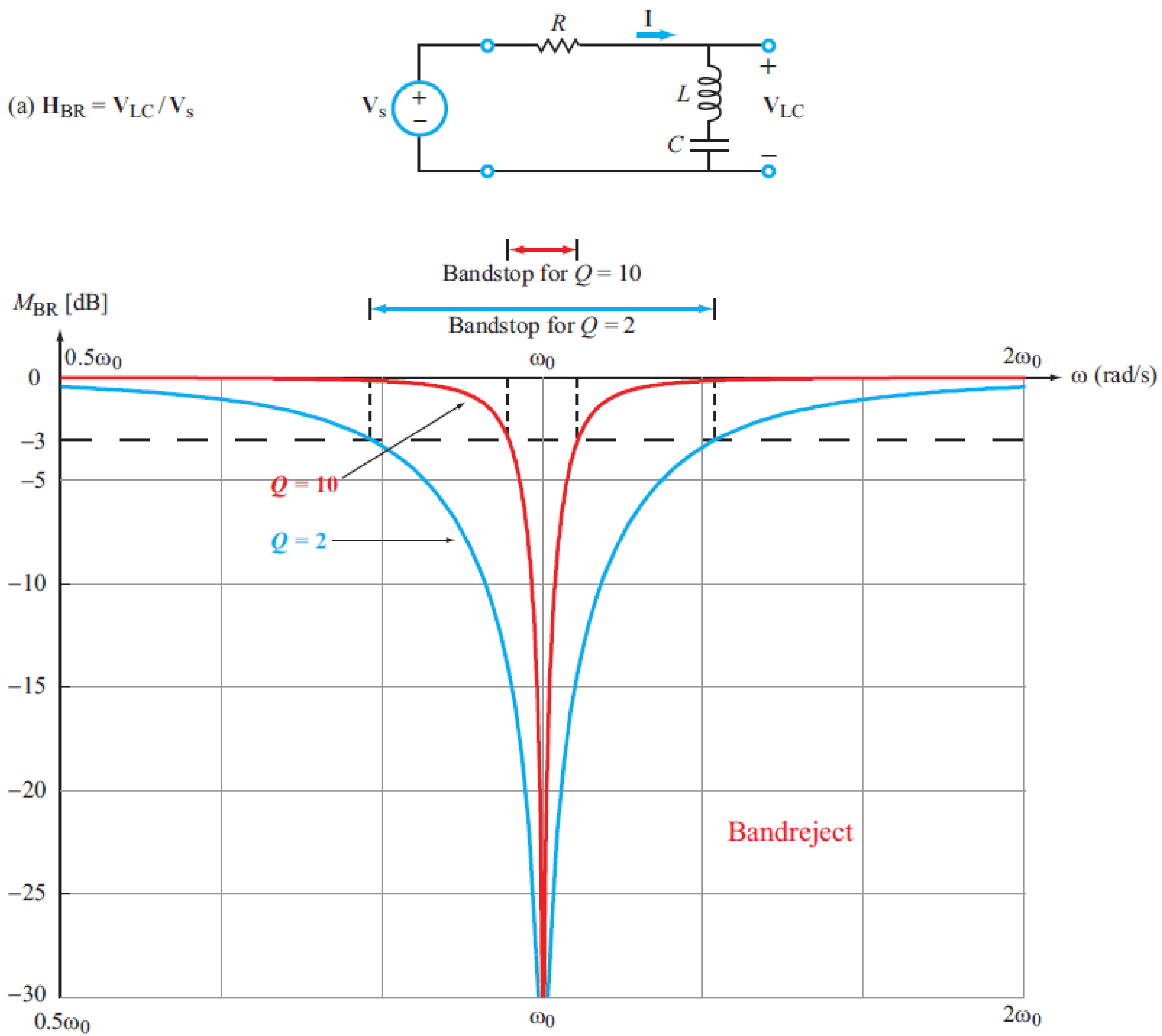
Lowpass Filter



(a) $H_{LP} = V_C / V_s$



Bandreject



(a) H_BR = V_LC / V_s
R
I
V_s
+
−
L
C
+
V_LC
−
Bandstop for Q = 10
Bandstop for Q = 2
M_BR [dB]
0.5ω0
ω0
2ω0
ω (rad/s)
0
−3
−5
−10
−15
−20
−25
−30
Q = 10
Q = 2
Bandreject
0.5ω0
ω0
2ω0

Filter Order