Discussion 5B

1. DFT

(a) Compute the DFT coefficients of $x_1[t] = \cos(\frac{2\pi}{6}t)$ where $t \in \{0, 1, \ldots, 5\}$.

(b) Plot the magnitude and phase for both time-domain and DFT-basis representations of $\vec{x}_1$.

(c) Compute the DFT coefficients of $x_2[t] = \cos(\frac{4\pi}{6}t)$ where $t \in \{0, 1, \ldots, 5\}$.

(d) Plot the magnitude and phase for both time-domain and DFT-basis representations of $\vec{x}_2$.

(e) How about the general case, $x_k[t] = \cos(\frac{2\pi}{6}kt)$, where $t \in \{0, 1, \ldots, 5\}$?

(f) Compute the DFT coefficients of $\vec{s} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}^T$.

(g) Compute the DFT coefficients of $y_1[t] = \cos(\frac{2\pi}{6}t - \pi)$ where $t \in \{0, 1, \ldots, 5\}$.

(h) Consider an impulse response

$$\vec{h} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T.$$

Let $\vec{x}_1$ be the input to the LTI system characterized by $\vec{h}$. The output $\vec{z}$ is connected to $\vec{x}_1$ by $\vec{z} = C_{\vec{h}}\vec{x}_1$, where $C_{\vec{h}}$ is the circulant matrix that has $\vec{h}$ as its first column. What is $\vec{z}$? What is the relationship between $\vec{z}$, $\vec{x}_1$, and $\vec{y}_1$?

2. SVD

Compute the SVD of the following matrix.

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

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