1. DFT

(a) Compute the DFT coefficients of \( x_1[t] = \cos\left(\frac{2\pi}{6}t\right) \) where \( t \in \{0,1,\ldots,5\} \).

(b) Plot the magnitude and phase for both time-domain and DFT-basis representations of \( \vec{x}_1 \).

(c) Compute the DFT coefficients of \( x_2[t] = \cos\left(\frac{4\pi}{6}t\right) \) where \( t \in \{0,1,\ldots,5\} \).

(d) Plot the magnitude and phase for both time-domain and DFT-basis representations of \( \vec{x}_2 \).

(e) How about the general case, \( x_k[t] = \cos\left(\frac{2\pi}{6}kt\right) \), where \( t \in \{0,1,\ldots,5\} \)?

(f) Compute the DFT coefficients of \( \vec{s} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}^T \).

(g) Compute the DFT coefficients of \( y_1[t] = \cos\left(\frac{2\pi}{6}t - \pi\right) \) where \( t \in \{0,1,\ldots,5\} \).

(h) Consider an impulse response

\[
\vec{h} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}^T.
\]

Let \( \vec{x}_1 \) be the input to the LTI system characterized by \( \vec{h} \). The output \( \vec{z} \) is connected to \( \vec{x}_1 \) by \( \vec{z} = C_{\hat{h}} \vec{x}_1 \), where \( C_{\hat{h}} \) is the circulant matrix that has \( \vec{h} \) as its first column. What is \( \vec{z} \)? What is the relationship between \( \vec{z}, \vec{x}_1 \), and \( \vec{y}_1 \)?

2. SVD

Compute the SVD of the following matrix.

\[
A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}
\]

Contributors:

- Yen-Sheng Ho.
- Harrison Wang.