1. Two Capacitors

Consider the circuit below, assume that when \( t \leq 0 \), both capacitors have no charge ( \( V_1(t = 0) = 0 \) and \( V_2(t = 0) = 0 \)). At \( t = 0 \), the switch closes.

![Two Capacitor Circuit with Voltage Source](image)

(a) First, use Kirchoff’s Laws and the capacitor equation ( \( I = \frac{dV}{dt} \)) to find the differential equation of this circuit.

(b) As shown in class, we can write each voltage \( V(t) \) as \( V_{std}(t) + V_{trans}(t) \), where \( V_{std}(t) \) is the steady state function and \( V_{trans}(t) \) is the transient function.

What are the steady state functions of \( V_1(t) \) and \( V_2(t) \)?

(c) Now, replace \( V_1(t) = V_{1,\text{std}}(t) + V_{1,\text{trans}}(t) \) and \( V_2(t) = V_{2,\text{std}}(t) + V_{2,\text{trans}}(t) \) using the steady state functions you found in the previous part. Also, find the initial conditions of \( V_{1,\text{trans}}(t) \) and \( V_{2,\text{trans}}(t) \) when \( t = 0 \).

(d) Assume that \( C_1 = C_2 = 1 \), \( R_1 = \frac{1}{3} \), and \( R_2 = \frac{1}{2} \). Diagonalize the matrix \( A \) in \( \frac{d}{dt} \left( \begin{bmatrix} V_{1,\text{trans}}(t) \\ V_{2,\text{trans}}(t) \end{bmatrix} \right) = A \left( \begin{bmatrix} V_{1,\text{trans}}(t) \\ V_{2,\text{trans}}(t) \end{bmatrix} \right) \).

(e) Now that we have diagonalized the matrix, we can now work in the eigenspace. Let us call the transformed \( \begin{bmatrix} V_{1,\text{trans}}(t) \\ V_{2,\text{trans}}(t) \end{bmatrix} \) as \( \tilde{V}(t) = \begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix} \). Solve for \( \tilde{V}(t) \). Do not forget about the initial conditions of \( \tilde{V}(t) \).

(f) Now that we have \( \tilde{V}(t) \), find the solution for \( \begin{bmatrix} V_{1,\text{trans}}(t) \\ V_{2,\text{trans}}(t) \end{bmatrix} \).

(g) For the final step, solve for \( \begin{bmatrix} V_1(t) \\ V_2(t) \end{bmatrix} \).

(h) Sketch the voltage vs time plots of \( V_1(t) \) and \( V_2(t) \).
2. RC Circuit - NAND  

Let us consider the RC Circuits of a NAND logic gate. This circuit implements the boolean function \( \neg (A \land B) \).

As shown in the figure, we can replace all the transistors with a resistor and switch but there is also a capacitor between the nMOS gates in the PUN. This capacitor complicates how long it takes for \( V_{out} \) to obtain the correct value. We will see how shortly.

Assume that \( A \) and \( B \) can only have two voltage values; \( V_{DD} \) (1) and Ground (0). \( A \) and \( B \) can change instantaneously.

(a) Assume for \( t < 0 \), \( A = 1 \) and \( B = 0 \) for a long enough time. What are the voltages across the two capacitors?
(b) At \( t = 0 \), the inputs change: \( A = 1 \) and \( B = 1 \). What is the resulting circuit of the NAND? Find the differential equation for \( V_{out} \).
(c) Assume that \( R_1 = \frac{1}{3} \), \( R_2 = 1 \), \( C = \frac{1}{2} \), and \( C_L = \frac{1}{5} \). Solve the differential equation in part (b).
(d) Let us try another situation. Assume for \( t < 0 \), \( A = 0 \) and \( B = 1 \). At \( t = 0 \), the inputs change: \( A = 1 \) and \( B = 1 \). What is the resulting circuit of the NAND? What are the initial conditions for the voltages across the two capacitors? Find the differential equation for \( V_{out} \).
(e) Assume that \( R_1 = \frac{1}{3} \), \( R_2 = 1 \), \( C = \frac{1}{2} \), and \( C_L = \frac{1}{3} \). Solve the differential equation in part (d).

Contributors:

- Lev Tauz.
- Varun Mishra.
Figure 2: NAND

Figure 3: RC Model of NAND