1. **RLC circuit**  In this problem, we study the differential equations governing a series RLC circuit, which we solve to get the transient behavior. We consider the simple RLC circuit below. Suppose that the switch is closed at time \( t = 0 \).

\[
\begin{align*}
\frac{d}{dt} \begin{pmatrix} i \\ v_L \end{pmatrix} &= A \begin{pmatrix} i \\ v_L \end{pmatrix} \\
\text{where } A &= \begin{pmatrix} R & -L \\ C & -R \end{pmatrix}
\end{align*}
\]

(a) Write the voltages \( v_C, v_R, v_L \) in terms of the current \( i \), with respect to time \( t \).

(b) Write down a second order differential equation for the current in the circuit with respect to time, in terms of the constants \( R, L, C \).

(c) Rewrite your second order differential equation in the form

\[
\begin{pmatrix} i \\ v_L \end{pmatrix} = c_0 e^{\lambda_0 t} v_0 + c_1 e^{\lambda_1 t} v_1,
\]

where \( v_0, v_1 \) are the eigenvectors of \( A \) with eigenvalues \( \lambda_0, \lambda_1 \) respectively.

(d) Find the eigenvalues and corresponding eigenvectors of your matrix \( A \) from the previous part.

(e) For the case where the two eigenvalues are real, we claim that the solution to this system of differential equations is of the form

\[
\begin{pmatrix} i \\ v_L \end{pmatrix} = c_0 e^{\lambda_0 t} v_0 + c_1 e^{\lambda_1 t} v_1,
\]

where \( c_0, c_1 \) are constants, and \( \lambda_0, \lambda_1 \) are the eigenvalues of \( A \) with eigenvectors \( v_0, v_1 \) respectively. Solve for the constants \( c_0, c_1 \), with the initial conditions \( i = 0, v_L = 1 \) at \( t = 0 \). Write your solution for \( i \) as a function of \( t \).

(f) For the case where the eigenvalues are complex, the solution to the system has the same form as in the previous part. Find \( i \) as a function of \( t \) in this case.

2. **RLC circuit in AC**

We study a simple RLC circuit with an AC voltage source given by

\[
v_s = B \cos(\omega t - \phi)
\]
(a) Write out the phasor representation of $v_s, R, C, L$.
(b) Use Kirchhoff’s laws to write down a loop equation relating the phasors in the previous part.
(c) Solve the equation in the previous step for the current $I$. What is the polar form of $I$?
(d) Compute the polar form of $V_R, V_L, V_C$.

Contributors:

- Lynn Chua.