1. **State Feedback - Pole Placement**

   Consider the following linear discrete time system

$$\vec{x}[t+1] = \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} \vec{x}[t] + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u[t]$$

   (a) Is this system controllable?

   (b) Is the linear discrete time system stable?

   (c) Derive a state space representation of the resulting closed loop system using state feedback of the form $u[t] = -\begin{bmatrix} f_1 & f_2 \end{bmatrix} \vec{x}[t]$

   (d) Find the appropriate state feedback constants, f_1, f_2 in order the state space representation of the resulting closed loop system to place the eigenvalues at $\lambda_1 = -\frac{1}{2}, \lambda_2 = \frac{1}{2}$

   (e) Is the system now stable?

2. **State Feedback - Eigenvalue Placement vs Observability** Consider the following linear discrete time system

$$\vec{x}[t+1] = \begin{bmatrix} 10 & 2 \\ -16 & -2 \end{bmatrix} \vec{x}[t] + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u[t]$$
$$\vec{y} = \begin{bmatrix} 1 & 0 \end{bmatrix} \vec{x}$$

   (a) Is this system controllable?

   (b) Is this system observable?

   (c) Is the linear discrete time system stable on its own if we don't apply an input?

   (d) Derive a state space representation of the resulting closed loop system using state feedback of the form $u[t] = \begin{bmatrix} 10 & 2 \end{bmatrix} \vec{x}[t]$ and prove that it is stable

   (e) Is the closed loop system using state feedback of the form $u[t] = \begin{bmatrix} 10 & 2 \end{bmatrix} \vec{x}$ observable?

   (f) How can select state feedback that stabilizes, but avoids losing observability?

   (g) Find a state feedback law such that the system is both stable and observable?

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