Resonant Circuits and MEMS

Consider the equivalent circuit of a crystal oscillator:

\[ i_1 + i_2 = i_f \]

This equation can be expressed in terms of voltages and capacitances:

\[
\frac{V_1}{j\omega C_1} + \frac{V_2}{j\omega C_2} = -\frac{V_{out}}{j\omega C_f}
\]

\[
\rightarrow -\left(\frac{C_1}{C_f}V_1 + \frac{C_2}{C_f}V_2\right) = V_{out}
\]

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If we say that some $C_o$ is the average of $C_1$ and $C_2$, then,

\[ C_1 = C_o + \Delta C \]
\[ C_2 = C_o - \Delta C \]

Our equation for $V_{out}$ then becomes:

\[ \left( \frac{C_o + \Delta C}{C_f} \right) V_1 - \left( \frac{C_o - \Delta C}{C_f} \right) V_2 = V_{out} \]

\[ \rightarrow - \frac{C_o}{C_f} V_1 - \frac{\Delta C}{C_f} V_1 - \frac{C_o}{C_f} V_2 + \frac{\Delta C}{C_f} V_2 = V_{out} \]

If we choose $V_1 = -V_2$ and say that this voltage is $V_{in}$, then the first and third terms cancel each other out, so then the equation for $V_{out}$ becomes:

\[ V_{out} = -\frac{2V_{in}}{C_f} \Delta C \]