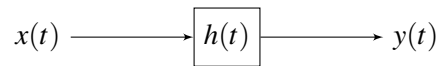


Notes

Linear Time-Invariant (LTI) Systems

We will represent a system as follows, with $x(t)$ as an input into system $h(t)$ and $y(t)$ as the output. In next week's section, we will learn how to characterize a system in order to find $h(t)$, called the **impulse response**. This week, we will focus on how the properties of **linearity** and **time invariance** affect the inputs and outputs, $x(t)$ and $y(t)$ of a system.



Linearity

A **linear system** has the properties below:

(a) **additivity**

$$x_1(t) + x_2(t) \longrightarrow \boxed{h(t)} \longrightarrow y_1(t) + y_2(t) \quad (1)$$

(b) **scaling**

$$\alpha x(t) \longrightarrow \boxed{h(t)} \longrightarrow \alpha y(t) \quad (2)$$

Together, these two properties are known as **superposition**:

$$\alpha_1 x_1(t) + \alpha_2 x_2(t) \longrightarrow \boxed{h(t)} \longrightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

Time invariance

A system is **time-invariant** if its behavior is fixed over time:

$$x(t - t_0) \longrightarrow \boxed{h(t)} \longrightarrow y(t - t_0) \quad (3)$$

Questions

1. Time-Shift Systems

Imagine we have a system $S_{\rightarrow 2}$ that takes any length 5 input signal and shifts it by 2 steps. For example, $S_{\rightarrow 2}([3, 1, 4, 1, 5]) = [1, 5, 3, 1, 4]$.

- Is this system linear? That is, for any signals \vec{x} and \vec{y} , does $S_{\rightarrow 2}$ fulfill properties (1) and (2)?
- Is this system time-invariant? Does it fulfil (3)?
- What does $S_{\rightarrow 2}$ look like when written as a matrix?

2. Is it LTI?

Determine if the following systems are LTI:

- $y[t] = 2x[-2 + 3t] + 2x[2 + 3t]$
- $y[t] = 4^{x[t]}$
- $y[t] - y[t - 1] + y[t - 2] = x[t] - x[t - 1] - x[t - 2]$
- $y[t] = x[t] - x[t - 2]$
- $y[t] = x[t] + tx[t - 1]$
- $y[t] = 2^t \cos(x[t])$

3. The DFT basis and LTI systems

Suppose \vec{x} is the input signal applied to a linear time-invariant (LTI) system characterized by the impulse response \vec{h} . The output \vec{y} is given by $C_{\vec{h}}\vec{x}$ where $C_{\vec{h}}$ is a circulant matrix with the first column given by \vec{h} . Suppose the DFT coefficients of \vec{x} are given by

$$\vec{X} = [X[0] \ X[1] \ \dots \ X[n-1]]^T.$$

- Compute the DFT representation of the impulse response \vec{h} as well as the eigenvalues of the circulant matrix that defines the system with that impulse response. ($n = 3$)

$$\vec{h} = [1 \ 0 \ 0]^T$$

- Compute the DFT representation of the impulse response \vec{h} as well as the eigenvalues of the circulant matrix that defines the system with that impulse response. ($n = 3$)

$$\vec{h} = [\cos(\frac{2\pi}{3}(0)) \ \cos(\frac{2\pi}{3}(1)) \ \cos(\frac{2\pi}{3}(2))]^T$$

- Compute the DFT representation of the impulse response \vec{h} as well as the eigenvalues of the circulant matrix that defines the system with that impulse response. ($n = N$)

$$\vec{h} = [\cos(\frac{2\pi k}{N}(0)) \ \cos(\frac{2\pi k}{N}(1)) \ \dots \ \cos(\frac{2\pi k}{N}(n-1))]^T$$

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