Euler's Formula

The following relationship is very useful and will be used in detail later in the course. For now, it will be useful for one of the questions.

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

1. Solutions of Second Order Differential Equations

Consider a differential equation of the form,

$$\frac{d^2 f}{dt^2}(t) + a_1 \frac{df}{dt}(t) + a_0 f(t) = 0$$

such that,

$$f(t) = c_1 e^{\lambda t} + c_2 e^{\overline{\lambda} t}$$

where $f(\cdot)$ is a real valued function from $\mathbb{R}$ to $\mathbb{R}$.

(a) Use the fact that f is real to prove that c_1 and c_2 are complex conjugates of each other.
Hint. Let $c_1 = a_1 + jb_1, c_2 = a_2 + jb_2$ and $\lambda = \sigma + j\omega$.

(b) Let $c = a + jb$ and $\lambda = \sigma + j\omega$. Show that you can reduce $f(t)$ to the following form:

$$f(t) = \left(2a\cos(\omega t) - 2b\sin(\omega t)\right) e^{\sigma t}$$

(c) When solving for the original differential equation, why do we not need to solve for c_1 and c_2 and instead we directly jump to a and b?

(d) What happens if $\sigma < 0$?

(e) What happens if $\sigma = 0$?

(f) What happens if $\sigma > 0$?