Notes

Constant External Force Nonhomogeneous Differential Equations

The following differential equation is a nonhomogeneous, constant external force differential equation:

$$\frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b$$

where b is a constant.

Even though this expression isn't equal to 0, we can still solve it using our method for homogeneous differential equations. If we substitute y with $\tilde{y} = y - \frac{b}{a_0}$, then we end up with a new differential equation that is homogeneous:

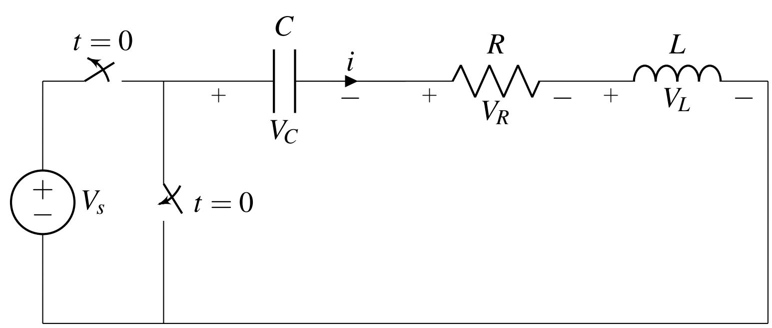
$$\frac{d^2\tilde{y}}{dt^2} + a_1 \frac{d\tilde{y}}{dt} + a_0 \tilde{y} = 0$$

Now we can solve for $\tilde{y}$ and then reverse our substitution to get y.

Questions

1. RLC circuit

Consider the following circuit:



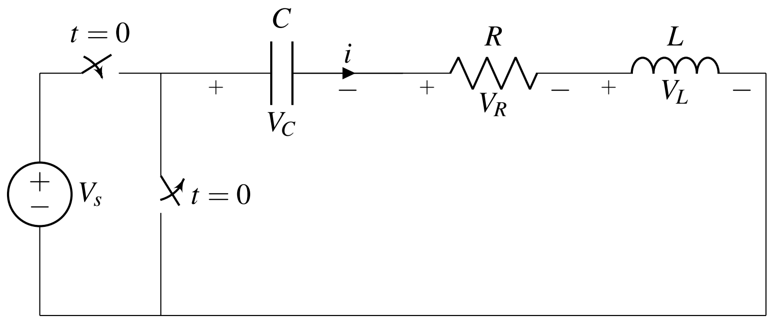
(a) Draw the circuit corresponding to $t < 0$. What are the values of V_C, V_R, V_L, and i at $t = 0_-$, the time right before the switches close. Assume this circuit has been in this state for a long time.

(b) Now draw the circuit corresponding to $t \geq 0$. Using your results from the previous part, what are V_C, V_R, V_L, and i at $t = 0_+$.

(c) Assuming the solution of the differential equation for V_C has the form $V_C(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$, what are the values of c_1 and c_2? Treat λ_1 and λ_2 as known constants.

2. Charging RLC Circuit

Consider the following circuit:



(a) Write out the differential equation describing this circuit for $t \geq 0$ in the form:

$$\frac{d^2V_c}{dt^2} + a_1\frac{dV_c}{dt} + a_0V_c = b$$

(b) Find a $\tilde{V}_c$ and substitute it to the previous equation such that

$$\frac{d^2\tilde{V}_c}{dt^2} + a_1\frac{d\tilde{V}_c}{dt} + a_0\tilde{V}_c = 0$$

(c) Solve for $\tilde{V}_c$.

(d) Solve for V_C

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