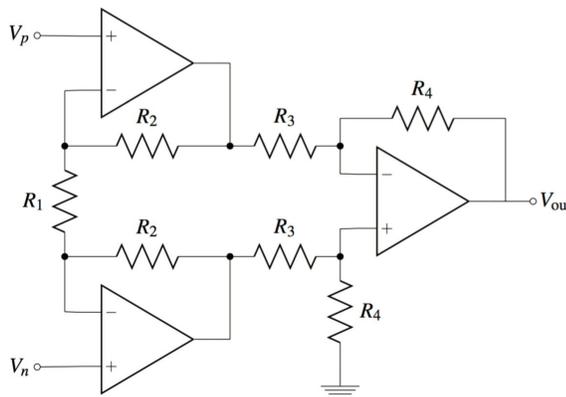


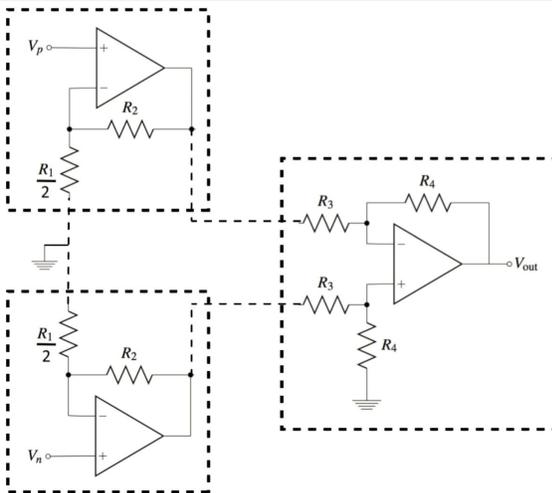
**This homework is due January 25, 2017, at 17:00.**

**1. Op-amp review**

Let us look at the circuit below:



- Write down all the branch and node equations using the golden rules of Op-Amps.
- Notice that there exists a symmetry between the two op-amps at the first stage of this circuit. What are the directions of the currents going through the two  $R_2$ s? How do the currents of  $R_2$ s influence the current through  $R_1$ ?
- What is the current through  $R_1$ ?
- What are the output voltages of the two op-amps at the first stage?
- Compute the voltage at the + terminal of the second-stage op-amp.
- What is  $V_{out}$ ?
- If we broke  $R_1$  into two series resistors, each with a value of  $\frac{R_1}{2}$ . What is the voltage at the node in between each of these resistors?
- Based on the above analysis, we could introduce a "fake ground" in the middle of the resistor  $R_1$  and come up with the following circuit:



Now, each of the first two op-amps is being used in a form that resembles building blocks that you have seen before. What are the gains of those blocks?

What is  $V_{out}/(V_p - V_n)$  for this revised circuit?

## 2. KVL

Now consider the circuit shown below:

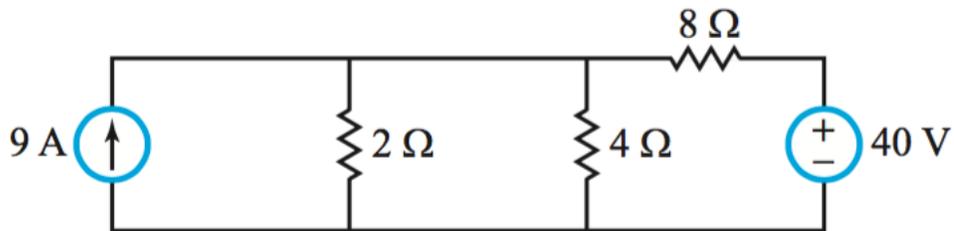


Figure 1: From Ulaby, Maharbiz, Furse. *Circuits*. Third Edition.

Determine the amount of power supplied by the voltage source.

## 3. KCL

Now consider the circuit shown below:

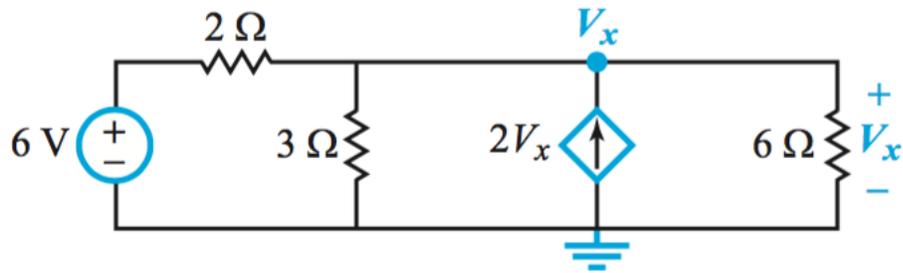


Figure 2: From Ulaby, Maharbiz, Furse. *Circuits*. Third Edition

Determine the voltage  $V_x$ .

#### 4. Circuits and gaussian elimination

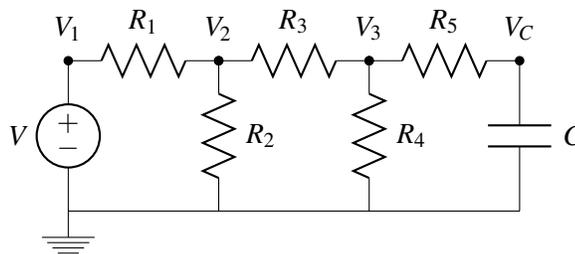


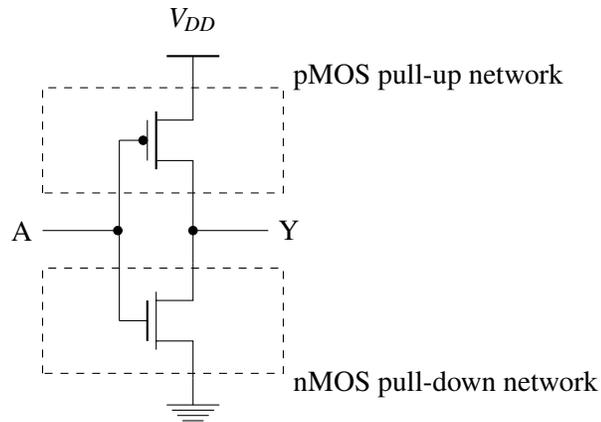
Figure 3: Example Circuit

- Find a system of linear equations that could be solved to find the node voltages.
- Given that the component values are  $R_1 = 500\Omega$ ,  $R_2 = 3000\Omega$ ,  $R_3 = 1000\Omega$ ,  $R_4 = 2000\Omega$ , and  $R_5 = 4000\Omega$ , solve the circuit equations using Gaussian elimination.
- What's the voltage  $V_C$  across the capacitor?
- How would you check your work? Do so.

#### 5. Transistors and boolean logic

A Boolean formula can be implemented in digital circuitry using nMOS and pMOS transistors. In circuits, the truth value 1 (*true*) is represented by a high voltage, called POWER ( $V_{DD}$ ). The truth value 0 (*false*) is represented by a low voltage, called GROUND (GND). In this problem, we will only use the truth values in order to simplify notations. That is, if you see  $A = 1$  for a point  $A$ , then it means the voltage of  $A$  is equal to  $V_{DD}$ . Similarly, if  $A = 0$ , then the voltage of  $A$  is equal to GND.

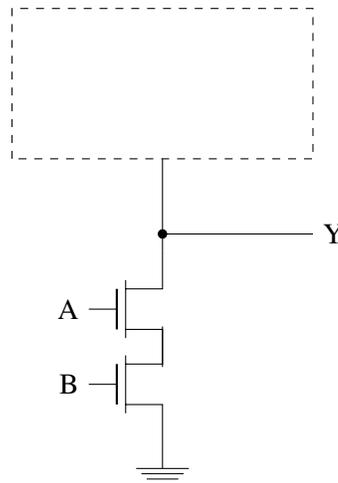
An inverter can be implemented with 1 nMOS and 1 pMOS, as shown in the figure below. When the input  $A$  is 0, then the nMOS is OFF and the pMOS is ON. Thus, the output  $Y$  is pulled up to 1 because it is connected to  $V_{DD}$ . Conversely, when  $A$  is 1, then the nMOS is ON and the pMOS is OFF, and  $Y$  is pulled down to 0. Therefore, the circuit implements the Boolean formula,  $Y = \neg A$ .



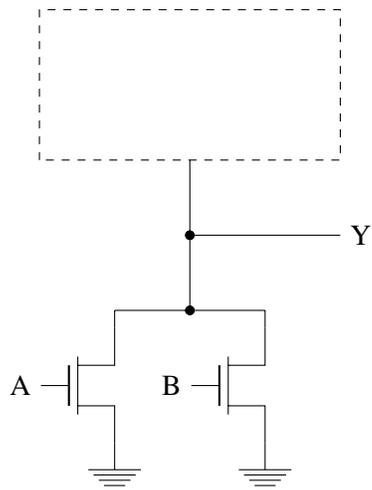
In general, a Boolean-formula circuit has an nMOS *pull-down network* to connect the output to 0 (GND) and a pMOS *pull-up network* to connect the output to 1 ( $V_{DD}$ ). The pull-up and pull-down networks in the inverter example each consist of a single transistor.

In this problem, we will ask you to design pull-up networks when pull-down networks are given.

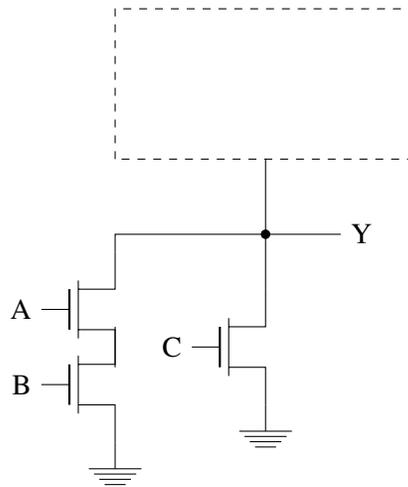
- (a) The pull-down network of the Boolean formula (a 2-input NAND gate),  $Y = \neg(A \wedge B)$ , is given below. Design the pull-up network (the dashed box) with 2 pMOS transistors.



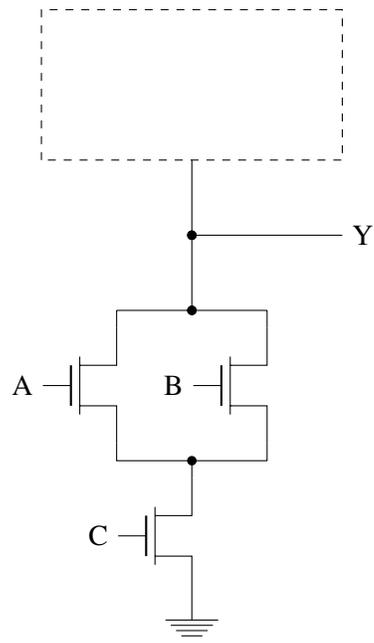
- (b) The pull-down network of the Boolean formula (a 2-input NOR gate),  $Y = \neg(A \vee B)$ , is given below. Design the pull-up network (the dashed box) with 2 pMOS transistors.



- (c) The pull-down network of the Boolean formula,  $Y = \neg((A \wedge B) \vee C)$ , is given below. Design the pull-up network (the dashed box) with 3 pMOS transistors.



- (d) The pull-down network of the Boolean formula,  $Y = \neg((A \vee B) \wedge C)$ , is given below. Design the pull-up network (the dashed box) with 3 pMOS transistors.



(e) You have designed four pull-up networks. What can you conclude about the rules for designing pull-up networks when pull-down networks are given?

**Contributors:**

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