This homework is due February 22, 2017, at 17:00.

1. **Bandpass filter** Consider the series bandpass filter below where $\tilde{V}_s$ and $\tilde{V}_o$ are phasor voltages:

![Bandpass Filter Diagram]

(a) What is the transfer function, $H(\omega) = \frac{\tilde{V}_o}{\tilde{V}_s}$, of this circuit?

(b) What is $\omega_0$ of this filter?

(c) What is $\omega_c_1$ and $\omega_c_2$ of this filter? Hint: $H(\omega_c_1) = H(\omega_c_2) = \frac{1}{\sqrt{2}}$

(d) What is the bandwidth, $B$, of this filter?

(e) What is the Q of this filter?

2. **Bode plots**

(a) **Transfer Functions to Bode Plots**

Problem 9.18: Generate Bode magnitude and phase plots (straight-line approximation) for the following voltage transfer functions:

- (a) $H(\omega) = \frac{30(10 + j\omega)}{(200 + j2\omega)(1000 + j2\omega)}$
- (b) $H(\omega) = \frac{j100\omega}{(100 + j5\omega)(100 + j\omega)\omega}$
- (c) $H(\omega) = \frac{200 + j2\omega}{(50 + j5\omega)(1000 + j\omega)}$

(b) Bandstop
3. **Ring oscillator**  Figure 4 shows a ring oscillator circuit with three inverters. These inverters are modeled as non-ideal op-amps using a general, non-ideal, op-amp model. Remember, **our golden rules don’t apply** for the models below. Each op-amp acts as an inverter with gain. The voltage inputs terminals are considered open circuits. $R_{out} = 10k\Omega$, $C_{o1} = C_{o2} = C_{o3} = 1pF$, and $K_1 = K_2 = K_3 = 2$
Figure 4: Ring Oscillator Modeled with Non-Ideal Op-Amps

(a) First, let’s look at the first op-amp in the chain. For the circuit in figure 5, find the transfer function for \( \frac{\bar{v}_1}{v_0} \).

Figure 5: First Op-Amp in Ring Oscillator

(b) Now, let’s look at three of these op-amps cascaded together as seen in figure 6. What is the transfer function for \( \frac{\bar{v}_1}{v_0} \)? (Hint: since the input of each op-amp is an open circuit, the overall transfer function can be represented as the individual transfer functions of each amplifier cascaded together.)

Figure 6: Ring Oscillator without Feedback
(c) Draw the bode plots for the magnitude and phase of \( \tilde{v}_3 / \tilde{v}_0 \).
(d) At what frequency is the phase of \( \tilde{v}_3 \) equal to \(-2\pi\)? What is the magnitude of \( \tilde{v}_3 / \tilde{v}_0 \) at that frequency? How does \( \tilde{v}_3 \) compare to \( \tilde{v}_0 \) at this frequency? An interesting consequence of this result is that this system will have a sustained oscillation when placed in feedback.

4. Redo Problem 1 on the midterm

(a)
(b)
(c)
(d)

5. Redo Problem 2 on the midterm

(a)
(b)
(c)
(d)

6. Redo Problem 3 on the midterm

(a)
(b)

7. Redo Problem 4 on the midterm

(a)
(b)
(c)
(d)
(e)

8. Redo Problem 5 on the midterm

(a)
(b)
Important Instructions:

Show your work. An answer without explanation is not acceptable and does not guarantee any credit.

Only the front pages will be scanned and graded. Back pages won't be scanned; you can use them as scratch paper.

Do not remove pages, as this disrupts the scanning. Instead, cross the parts that you don't want us to grade.

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“Well, Diotallevi and I are planning a reform in higher education. A School of Comparative Irrelevance, where useless or impossible courses are given. The school’s aim is to turn out scholars capable of endlessly increasing the number of unnecessary subjects.”  
— Umberto Eco, *Foucault’s Pendulum*

**Problem 1** *Warm up* (20 points)

a) Consider the following circuit. $Z_{eq}$ is the impedance looking into the circuit from the left, as shown. Provide an expression for $Z_{eq}$.

\[ Z_{eq} = \]
b) If this impedance is driven by a sinusoidal source at frequency, \( \omega \) [rad/s], for what \( \omega \) is \( Z_{eq} = \infty \)?

\[ \omega = \]

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c) What logic function does the following circuit perform?
d) Consider the following four circuits. For each, we define the voltage transfer function, \( H_v(\omega) = \frac{V_{out}}{V_{in}} \).
With respect to \( H_v(\omega) \), circle what class of frequency response each circuit performs.

- Lowpass filter
- Highpass filter
- Bandpass filter
- Bandstop filter

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**Circuit 1**
- Lowpass filter
- Highpass filter
- Bandpass filter
- Bandstop filter

**Circuit 2**
- Lowpass filter
- Highpass filter
- Bandpass filter
- Bandstop filter

**Circuit 3**
- Lowpass filter
- Highpass filter
- Bandpass filter
- Bandstop filter

**Circuit 4**
- Lowpass filter
- Highpass filter
- Bandpass filter
- Bandstop filter
Problem 2 (20 points)

Consider the circuit below. Assume an ideal op amp.

a) Find an expression that relates the derivative of $v_{out}$ ($\frac{dv_{out}}{dt}$) to the input voltage ($v_{in}$) and/or its derivative ($\frac{dv_{in}}{dt}$).

\[
\frac{dv_{out}}{dt} =
\]

b) Now given that $C_s = 1 \text{ nF}$, $C_f = 5 \text{ nF}$, $C_p = 1 \text{ nF}$, $v_{Cf}(t<0) = v_{Cs}(t<0) = 0$ and $v_{in}(t\geq0) = 5*t \text{ [volts]}$, provide an expression for $v_{out}(t)$ for $t\geq0$. 

"You can tell you've found a really interesting question when nobody wants you to answer it."
― James S.A. Corey, *Nemesis Games*
Consider now the different circuit below. Assume an ideal op amp.

\[ v_{\text{out}}(t) = \]

\[ d) \text{ Assume } v_{\text{in}}(t \geq 0) = 5t \text{ [volts]} \] and \[ v_{\text{cf}}(t < 0) = 0 \]. What is the value of \[ v_{\text{out}}(t) \] at \( t = 1 \text{ s} \)?

\[ v_{\text{out}}(t) = \]
Problem 3 (15 points)

The following circuit is part of a near field communication system. A realistic voltage source ($V_s, R_s$) is connected through a switch onto a three component circuit. The inductor represents an antenna; the voltage across it modulates how much energy is radiated away from the system. The switch alternates continuously between position A and position B; it has been doing this since $t = -\infty$. It spends $\pi$ microseconds at each position.

We want the voltage on the inductor, $V_L$, to follow the curve plotted below. Specifically, we want to fulfill the following condition.

**Condition:** The inductor voltage should oscillate 5 times during period when the switch is in position A.

Plot of $V_L$ as a function of time with switch positions labeled. Note the units of time ($10^{-6}$ seconds)!
a) If $R \to \infty$ and $L$ is non-zero and known, provide an expression for $C$ such that the above condition is met. (Reminder: the condition is that the inductor voltage should oscillate 5 times during period when the switch is in position A.)

\[ C = \]

b) Unfortunately, a colleague tells you that $R \neq \infty$; if $L$ and $C$ are known, provide an expression for $R$ such that the above condition is met. (Reminder: the condition is that the inductor voltage should oscillate 5 times during period when the switch is in position A.)

\[ R = \]
Problem 4 (30 points)

Consider the circuit below.

a) What is \( i(0) \)?
   
   *Hint. What is the current flowing through \( L1 \) before the switch opens? Consequently, what is the current flowing through \( L2 \)?*

b) What is \( \frac{di}{dt}(0) \)?

c) What is the relationship between the voltages across \( L1 \) and \( R1 \)?
d) Use KCL on Node A and the relationship derived above to arrive at a differential equation of the form,

\[ \frac{d^2i}{dt^2}(t) + a_1 \frac{di}{dt}(t) + a_0i(t) = 0 \]

where \(i(t)\) is the current going through L2.

e) Let \(R_1 = R_2 = R\) and \(L_1 = L_2 = L\). Recall that the above differential equation can be reshaped into the follow linear algebra problem:

\[
\begin{bmatrix}
\frac{di}{dt} \\
\frac{d^2i}{dt^2}
\end{bmatrix}
= A
\begin{bmatrix}
i \\
\frac{di}{dt}
\end{bmatrix}
\]

What is the A matrix and what are its eigenvalues?

f) Will this circuit exhibit any oscillations?
Problem 5 (15 points)

Consider the circuit below.

a) Given an input voltage, $v_1(t)$, which is a sinusoid at frequency $\omega$, and phasors corresponding to the input and output voltages, $V_1$ and $V_2$, find an expression for $V_2/V_1$.

\[
\frac{V_2}{V_1} = \text{expression here}
\]
b) If $v_1(t) = \cos(\omega t)$ where $\omega = 10^6 \text{ rad/s}$ and $L = 1 \mu\text{H}$, $R = 1 \Omega$, and $C = 0.5 \mu\text{F}$, solve for $v_2(t)$.

$$v_2(t) =$$
Contributors:

- Brian Kilberg.