1. Bandpass filter Consider the series bandpass filter below where $\tilde{V}_s$ and $\tilde{V}_o$ are phasor voltages:

(a) What is the transfer function, $H(\omega) = \frac{\tilde{V}_o}{\tilde{V}_s}$, of this circuit? **Solution:**

$$H(\omega) = \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}$$

(b) What is $\omega_0$ of this filter? **Solution:**

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\omega_0 = 10^4 \text{rads/s}$$

(c) What is $\omega_{\epsilon 1}$ and $\omega_{\epsilon 2}$ of this filter? Hint: $H(\omega_{\epsilon 1}) = H(\omega_{\epsilon 2}) = \frac{1}{\sqrt{2}}$ **Solution:**

$$\omega_{\epsilon 1} = 9875 \text{rads/s}$$

$$\omega_{\epsilon 2} = 10125 \text{rads/s}$$

(d) What is the bandwidth, $B$, of this filter? **Solution:** $B = 250 \text{rads/s}$

(e) What is the Q of this filter? **Solution:** $Q = 40$

2. Bode plots
(a) Transfer Functions to Bode Plots

**Problem 9.18** Generate Bode magnitude and phase plots (straight-line approximation) for the following voltage transfer functions:

(a) \( H(\omega) = \frac{30(10 + j\omega)}{(200 + j2\omega)(1000 + j2\omega)} \)

(b) \( H(\omega) = \frac{j100\omega}{(100 + j5\omega)(100 + j\omega)^2} \)

(c) \( H(\omega) = \frac{(200 + j2\omega)}{(50 + j5\omega)(1000 + j\omega)} \)

Figure 1: Reproduced with permission from Ulaby, Maharbiz, Furse. Circuits. Third Edition.

**Solution:**
Problem 9.18  Generate Bode magnitude and phase plots (straight-line approximation) for the following voltage transfer functions:

(a) \( H(\omega) = \frac{30(10 + j\omega)}{(200 + j2\omega)(1000 + j2\omega)} \)

(b) \( H(\omega) = \frac{j100\omega}{(100 + j5\omega)(100 + j\omega)^2} \)

(c) \( H(\omega) = \frac{(200 + j2\omega)}{(50 + j5\omega)(1000 + j\omega)} \)

Solution:

(a) 
\[
H(\omega) = \frac{30(10 + j\omega)}{(200 + j2\omega)(1000 + j2\omega)} = \frac{300(1 + j\omega/10)}{200 \times 1000(1 + j\omega/100)(1 + j\omega/500)}
\]

\( = \frac{1.5 \times 10^{-3}(1 + j\omega/10)}{(1 + j\omega/100)(1 + j\omega/500)} \)

\( \bullet \) Constant term \( 1.5 \times 10^{-3} \implies -56.5 \text{ dB} \)

\( \bullet \) Simple zero with \( \omega_c = 10 \text{ rad/s} \)

\( \bullet \) Simple pole with \( \omega_c = 100 \text{ rad/s} \)

\( \bullet \) Simple pole with \( \omega_c = 500 \text{ rad/s} \)
(b) \( H(\omega) = \frac{j100\omega}{(100 + j5\omega)(100 + j\omega)^2} = \frac{j10^{-4} \omega}{(1 + j\omega/20)(1 + j\omega/100)^2} \)

- Constant term \( 10^{-4} \implies -80 \text{ dB} \)
- Zero @ origin
- Simple pole with \( \omega_c = 20 \text{ rad/s} \)
- Simple pole with \( \omega_c = 100 \text{ rad/s}, \text{ of order 2} \)
\(M(\omega) = 20 \log \omega\)

\(\phi(\omega)\) with \(\omega_c = 10\ \text{rad/s}\)

(c) \(H(\omega) = \frac{(200 + j2\omega)}{(50 + j5\omega)(1000 + j\omega)} = \frac{(1 + j\omega/100)}{250(1 + j\omega/10)(1 + j\omega/1000)}\)

- Constant term \(1/250 \implies -48\ \text{dB}\)
- Simple pole with \(\omega_c = 10\ \text{rad/s}\)
- Simple zero with $\omega_c = 100$ rad/s
- Simple pole with $\omega_c = 1000$ rad/s

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(b) Bandstop

**Problem 9.22** Determine the voltage transfer function $H(\omega)$ corresponding to the Bode magnitude plot shown in Fig. P9.22. The phase of $H(\omega)$ is 90° at $\omega = 0$.

![Bode magnitude plot for Problem 9.22.](image)

Figure 2: Reproduced with permission from Ulaby, Maharbiz, Furse. Circuits. Third Edition.
Problem 9.22. Determine the voltage transfer function \( H(\omega) \) corresponding to the Bode magnitude plot shown in Fig. P9.22. The phase of \( H(\omega) \) is 90° at \( \omega = 0 \).

Solution: \( H(\omega) \) consists of:

1. A constant term \( K \) whose dB value is 60 dB, or
   \[ K = 10^{60/20} = 1000. \]
2. A simple pole of order 3 with \( \omega_c = 5 \text{ rad/s} \) (slope = −60 dB/decade)
3. A simple zero of order 6 with \( \omega_c = 50 \text{ rad/s} \) (slope reverses from −60 dB/decade to +60 dB/decade)
4. A simple pole of order 3 with \( \omega_c = 500 \text{ rad/s} \) (slope changes to 0 dB at \( \omega_c = 500 \text{ rad/s} \)).

Hence,

\[
H(\omega) = \frac{(j\omega)^6 1000(1 + j\omega/50)^6}{(1 + j\omega/5)^3(1 + j\omega/500)^3} = \frac{j1000(50 + j\omega)^6}{(5 + j\omega)^3(500 + j\omega)^3}.
\]

Given that the phase of \( H(\omega) \) is 90° at \( \omega = 0 \), it follows that \( N = 1 \).

Solution:

(c) Bandpass
**Problem 9.24** Determine the voltage transfer function $H(\omega)$ corresponding to the Bode magnitude plot shown in Fig. P9.24. The phase of $H(\omega)$ is $-90^\circ$ at $\omega = 0$.

![Bode magnitude plot](image)

Figure P9.24: Bode magnitude plot for Problem 9.24.

Figure 3: Reproduced with permission from Ulaby, Maharbiz, Furse. Circuits. Third Edition.
Problem 9.24  Determine the voltage transfer function $H(\omega)$ corresponding to the Bode magnitude plot shown in Fig. P9.24. The phase of $H(\omega)$ is $-90^\circ$ at $\omega = 0$.

![Bode magnitude plot for Problem 9.24.](image)

**Solution:** The transfer function consists of:

1. A simple zero of order $N$ with $\omega_z = 100\text{ rad/s}$
2. A simple pole of order $N$ with $\omega_p = 200\text{ rad/s}$
3. A simple pole of order $N$ with $\omega_p = 2000\text{ rad/s}$
4. A factor $-j$ (phase at $\omega = 0$ is $-90^\circ$).

Hence,

$$H(\omega) = \frac{-j(1 + j\omega/100)^N}{(1 + j\omega/200)^N(1 + j\omega/2000)^N}.$$

To determine $N$, we note that the first term reaches 36 dB at $\omega = 200\text{ rad/s}$. That is, the straight-line approximation

$$20\log |1 + \omega/100|^N \approx 20\log \left| \frac{\omega}{100} \right|_{\omega=200} = 36 \text{ dB},$$

or

$$20N \log 2 = 36 \text{ dB} \quad \implies \quad N = 6.$$

Hence,

$$H(\omega) = \frac{-j(200)^6(100 + j\omega)^6}{(100)^6(200 + j\omega)^6(2000 + j\omega)^6} = -j4.096 \times 10^2 \frac{(100 + j\omega)^6}{(200 + j\omega)^6(2000 + j\omega)^6}.$$

**Solution:**

3. **Ring oscillator**  Figure 4 shows a ring oscillator circuit with three inverters. These inverters are modeled as non-ideal op-amps using a general, non-ideal, op-amp model. Remember, our golden rules don’t apply for
the models below. Each op-amp acts as an inverter with gain. The voltage inputs terminals are considered open circuits. $R_{out} = 10k\Omega$, $C_{o1} = C_{o2} = C_{o3} = 1\ pF$, and $K_1 = K_2 = K_3 = 2$

Figure 4: Ring Oscillator Modeled with Non-Ideal Op-Amps
(a) First, let’s look at the first op-amp in the chain. For the circuit in figure 5, find the transfer function for \( \frac{\tilde{v}_1}{\tilde{v}_0} \).

\[
\tilde{v}_1 = K_1 v_{in1} \frac{Z_{C_{o1}}}{Z_{C_{o1}} + R_{o1}}
\]

\[
v_{in1} = -\tilde{v}_0
\]

Using the above equations, we get

\[
\frac{\tilde{v}_1}{\tilde{v}_0} = -\frac{K_1}{1 + j\omega R_{o1} C_{o1}}
\]

\[
\frac{\tilde{v}_1}{\tilde{v}_0} = -\frac{2}{1 + j\omega 10^8}
\]

(b) Now, let’s look at three of these op-amps cascaded together as seen in figure 6. What is the transfer function for \( \frac{\tilde{v}_3}{\tilde{v}_0} \)? (Hint: since the input of each op-amp is an open circuit, the overall transfer function can be represented as the individual transfer functions of each amplifier cascaded together.)

\[
\frac{\tilde{v}_3}{\tilde{v}_0} = \frac{\tilde{v}_1}{\tilde{v}_0} \cdot \frac{\tilde{v}_2}{\tilde{v}_1} \cdot \frac{\tilde{v}_3}{\tilde{v}_2}
\]
$$\frac{\tilde{v}_3}{\tilde{v}_0} = \frac{K_1 K_2 K_3}{(1 + j\omega R_0 C_0_1)(1 + j\omega R_0 C_0_2)(1 + j\omega R_0 C_0_3)}$$

Since all $K$, $R$, and $C$ are equal:

$$\frac{\tilde{v}_3}{\tilde{v}_0} = -\frac{K^3}{(1 + j\omega RC)^3}$$

$$\frac{\tilde{v}_3}{\tilde{v}_0} = -\frac{8}{(1 + j\frac{\omega}{10^3})^3}$$

(c) Draw the bode plots for the magnitude and phase of $\frac{\tilde{v}_3}{\tilde{v}_0}$.

**Solution:**

(d) At what frequency is the phase of $\frac{\tilde{v}_3}{\tilde{v}_0}$ equal to $-2\pi$? What is the magnitude of $\frac{\tilde{v}_3}{\tilde{v}_0}$ at that frequency? How does $\tilde{v}_3$ compare to $\tilde{v}_0$ at this frequency? An interesting consequence of this result is that this system will have a sustained oscillation when placed in feedback. **Solution:** We know that phase response adds when multiplied together (due to the exponential property of phase), so we need to find the $\omega$ where the contribution of each stage of the cascade is $-\frac{2\pi}{3}$, which leads to a total phase response of $-2\pi$ when we sum the phase response of all three inverters plus the sign inversion due to triple inversion. The total phase of our function is

$$\phi(\omega) = -\pi - 3 \cdot \text{atan}\left(\frac{\omega}{\omega_c}\right)$$
where \( \omega_c = \frac{1}{RC} \)

\[-2\pi = -\pi - 3 \ast \text{atan}\left( \frac{\omega}{\omega_c} \right)\]

\[\frac{\pi}{3} = \text{atan}\left( \frac{\omega}{\omega_c} \right)\]

\[\tan\left( \frac{\pi}{3} \right) = \frac{\sqrt{3}}{1} = \frac{\omega}{\omega_c}\]

\[\sqrt{3} \cdot \frac{1}{RC} = \omega\]

\[\omega = \sqrt{3} \ast 10^8\]

The magnitude of this at that \( \omega \) is

\[\frac{8}{(\sqrt{1 + 3})^3} = 1\]

As we can see, there is a certain frequency that causes the output voltage, \( \tilde{v}_3 \), to be a scaled version of the input signal \( \tilde{v}_0 \) after the phase shift and sign inversion. When the system is placed in feedback, these conditions cause the output of the system to reinforce the input of the system, leading to oscillation at the frequency where \( \phi(\omega) = -2\pi \).

4. Redo Problem 1 on the midterm

(a) **Solution:**

(b)

(c)

(d)

5. Redo Problem 2 on the midterm

(a)

(b)

(c)

(d)

6. Redo Problem 3 on the midterm

(a)

(b)

7. Redo Problem 4 on the midterm

(a)

(b)

(c)

(d)
8. **Redo Problem 5 on the midterm**

   (a)

   (b)
Important Instructions:

Show your work. An answer without explanation is not acceptable and does not guarantee any credit.

Only the front pages will be scanned and graded. Back pages won't be scanned; you can use them as scratch paper.

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“Well, Diotallevi and I are planning a reform in higher education. A School of Comparative Irrelevance, where useless or impossible courses are given. The school's aim is to turn out scholars capable of endlessly increasing the number of unnecessary subjects.”

— Umberto Eco, *Foucault's Pendulum*

**Problem 1 Warm up (20 points)**

a) Consider the following circuit. $Z_{eq}$ is the impedance looking into the circuit from the left, as shown. Provide an expression for $Z_{eq}$.

![Circuit Diagram]

\[ Z_{eq} = \]
b) If this impedance is driven by a sinusoidal source at frequency, \( \omega \) [rad/s], for what \( \omega \) is \( Z_{eq} = \infty \)?

\( \omega = \) 

c) What logic function does the following circuit perform?
d) Consider the following four circuits. For each, we define the voltage transfer function, \( H_v(\omega) = \frac{V_{out}}{V_{in}} \).
With respect to \( H_v(\omega) \), circle what class of frequency response each circuit performs.

- Lowpass filter
- Highpass filter
- Bandpass filter
- Bandstop filter
Problem 2 (20 points)

Consider the circuit below. Assume an ideal op amp.

![Circuit Diagram]

a) Find an expression that relates the derivative of $v_{out}$ ($dv_{out}/dt$) to the input voltage ($v_{in}$) and/or its derivative ($dv_{in}/dt$).

\[
\frac{dv_{out}}{dt} =
\]

b) Now given that $C_s = 1 \text{ nF}$, $C_f = 5 \text{ nF}$, $C_p = 1 \text{ nF}$, $v_{Cf}(t<0) = v_{Cs}(t<0) = 0$ and $v_{in}(t≥0) = 5*t \text{ [volts]}$, provide an expression for $v_{out}(t)$ for $t≥0$. 

“Vous pouvez dire que vous avez trouvé une question vraiment intéressante lorsque personne ne veut que vous la résolvez.”

— James S.A. Corey, *Nemesis Games*
Consider now the different circuit below. Assume an ideal op amp.

c) Provide a symbolic expression for \( V_{out}(t) \) for \( t \geq 0 \).

\[
V_{out}(t) =
\]

d) Assume \( V_{in}(t \geq 0) = 5t \ [\text{volts}] \) and \( V_{cf}(t < 0) = 0 \). What is the value of \( V_{out}(t) \) at \( t = 1 \ s \)?

\[
V_{out}(t) =
\]
Problem 3 (15 points)

The following circuit is part of a near field communication system. A realistic voltage source \((V_s, R_s)\) is connected through a switch onto a three component circuit. The inductor represents an antenna; the voltage across it modulates how much energy is radiated away from the system. The switch alternates continuously between position A and position B; it has been doing this since \(t = -\infty\). It spends \(\pi\) microseconds at each position.

![Circuit Diagram]

We want the voltage on the inductor, \(V_L\), to follow the curve plotted below. Specifically, we want to fulfill the following condition.

**Condition:** The inductor voltage should oscillate 5 times during period when the switch is in position A.

![Plot of V_L](image)

Plot of \(V_L\) as a function of time with switch positions labeled. **Note the units of time \(10^{-6}\) seconds!**
a) If $R \to \infty$ and $L$ is non-zero and known, provide an expression for $C$ such that the above condition is met. (Reminder: the condition is that the inductor voltage should oscillate 5 times during period when the switch is in position A.)

$$C =$$

b) Unfortunately, a colleague tells you that $R \neq \infty$; if $L$ and $C$ are known, provide an expression for $R$ such that the above condition is met. (Reminder: the condition is that the inductor voltage should oscillate 5 times during period when the switch is in position A.)

$$R =$$
“Chang Tzu tells us of a persevering man who after three laborious years mastered the art of dragon-slaying. For the rest of his days, he had not a single opportunity to test his skills.”
— Jorge Luis Borges, The Book of Imaginary Beings

Problem 4 (30 points)

Consider the circuit below.

a) What is \( i(0) \)?

*Hint. What is the current flowing through \( L_1 \) before the switch opens? Consequently, what is the current flowing through \( L_2 \)?*

b) What is \( \frac{di}{dt}(0) \)?

c) What is the relationship between the voltages across \( L_1 \) and \( R_1 \)?
d) Use KCL on Node A and the relationship derived above to arrive at a differential equation of the form,

$$\frac{d^2i}{dt^2}(t) + a_1\frac{di}{dt}(t) + a_0i(t) = 0$$

where $i(t)$ is the current going through L2.

e) Let $R_1 = R_2 = R$ and $L_1 = L_2 = L$. Recall that the above differential equation can be reshaped into the follow linear algebra problem:

$$\begin{bmatrix}
\frac{di}{dt} \\
\frac{d^2i}{dt^2}
\end{bmatrix} = A \begin{bmatrix}
i \\
\frac{di}{dt}
\end{bmatrix}$$

What is the A matrix and what are its eigenvalues?

f) Will this circuit exhibit any oscillations?
“I am Groot.”
- Groot, *Guardians of the Galaxy*

**Problem 5** (15 points)

Consider the circuit below.

![Circuit Diagram]

a) Given an input voltage, \( v_1(t) \), which is a sinusoid at frequency \( \omega \), and phasors corresponding to the input and output voltages, \( V_1 \) and \( V_2 \), find an expression for \( \frac{V_2}{V_1} \).

\[
\frac{V_2}{V_1} =
\]
b) If \( v_1(t) = \cos(\omega t) \) where \( \omega = 10^6 \text{ rad/s} \) and \( L = 1 \, \mu\text{H}, \, R = 1 \, \Omega, \, \text{and} \, C = 0.5 \, \mu\text{F} \), solve for \( v_2(t) \).

\[
v_2(t) = \]

Contributors:

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