

Problem 9.17 Generate Bode magnitude and phase plots (straight-line approximation) for the following voltage transfer functions:

(a) $H(\omega) = \frac{j100\omega}{10 + j\omega}$

(b) $H(\omega) = \frac{0.4(50 + j\omega)^2}{(j\omega)^2}$

(c) $H(\omega) = \frac{(40 + j80\omega)}{(10 + j50\omega)}$

(d) $H(\omega) = \frac{(20 + j5\omega)(20 + j\omega)}{j\omega}$

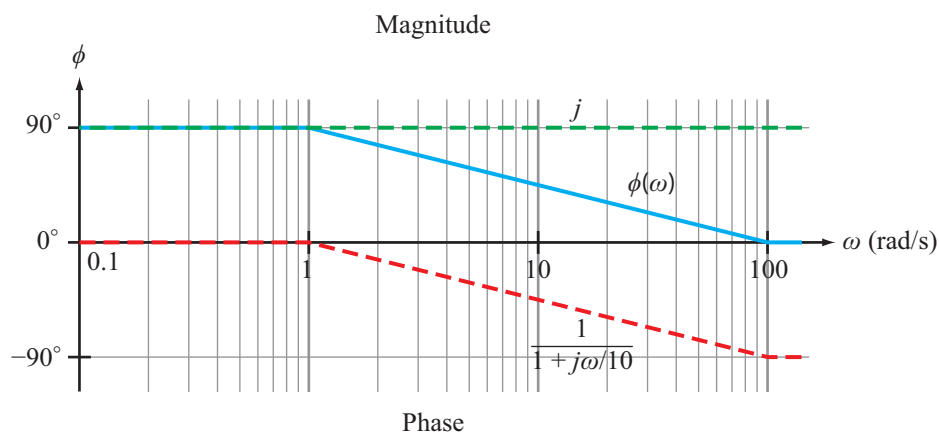
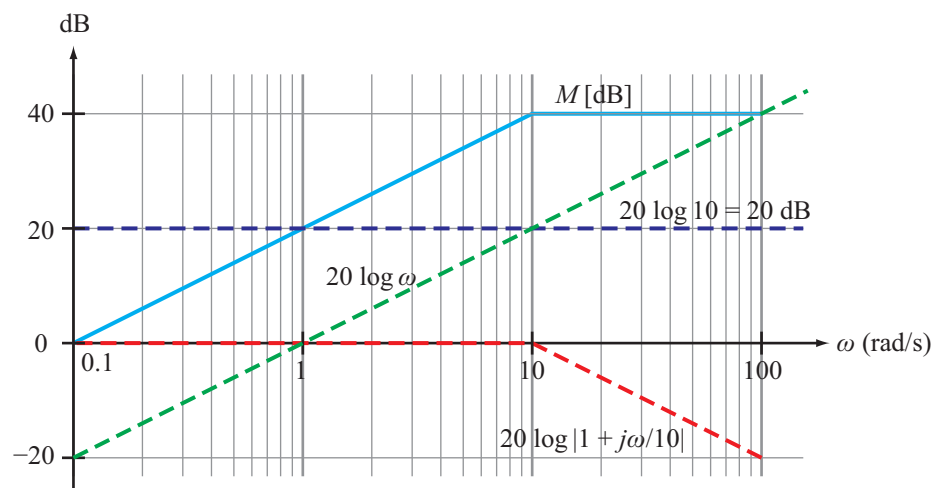
Solution:

(a) $H(\omega) = \frac{j100\omega}{10 + j\omega} = \frac{j100\omega}{10(1 + j\omega/10)} = \frac{j10\omega}{1 + j\omega/10}$

- Constant factor 10 $\Rightarrow +20$ dB
- Zero @ origin
- Simple pole with $\omega_c = 10$ rad/s

$$M [\text{dB}] = 20 \log |H|$$

$$= 20 \log 10 + 20 \log \omega - 20 \log |1 + j\omega/10|$$

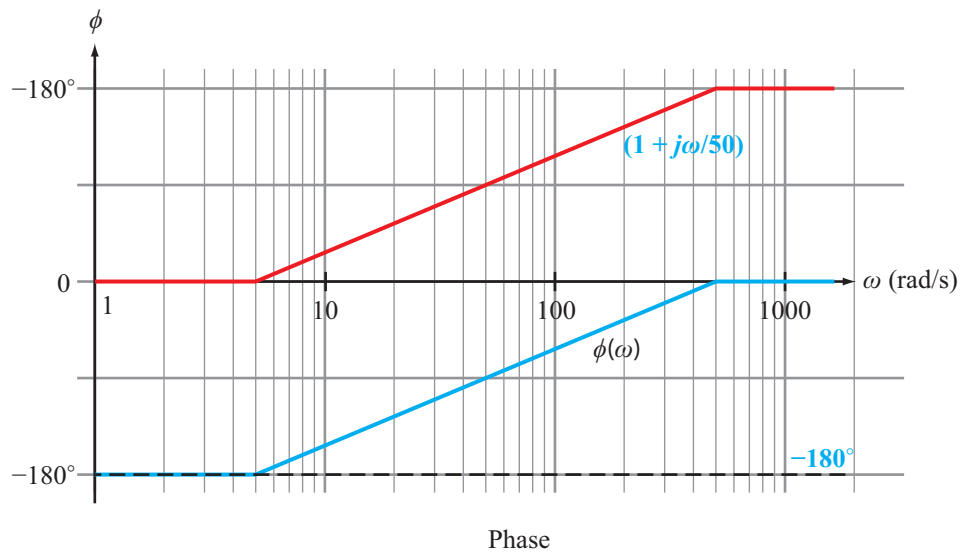
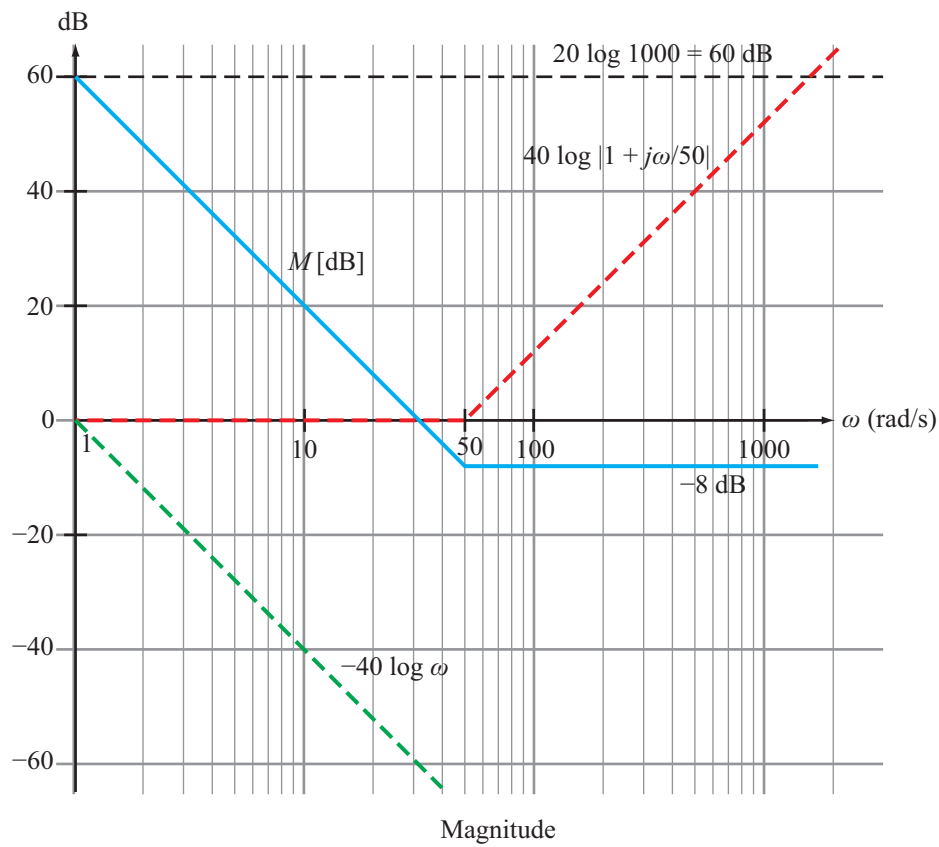


(b)

$$\begin{aligned}\mathbf{H}(\omega) &= \frac{0.4(50 + j\omega)^2}{(j\omega)^2} \\ &= \frac{0.4 \times 2500(1 + j\omega/50)^2}{\omega^2} = \frac{-1000(1 + j\omega/50)^2}{\omega^2}.\end{aligned}$$

$$M [\text{dB}] = 20 \log |\mathbf{H}(\omega)| = 20 \log 1000 + 40 \log |1 + j\omega/50| - 40 \log \omega.$$

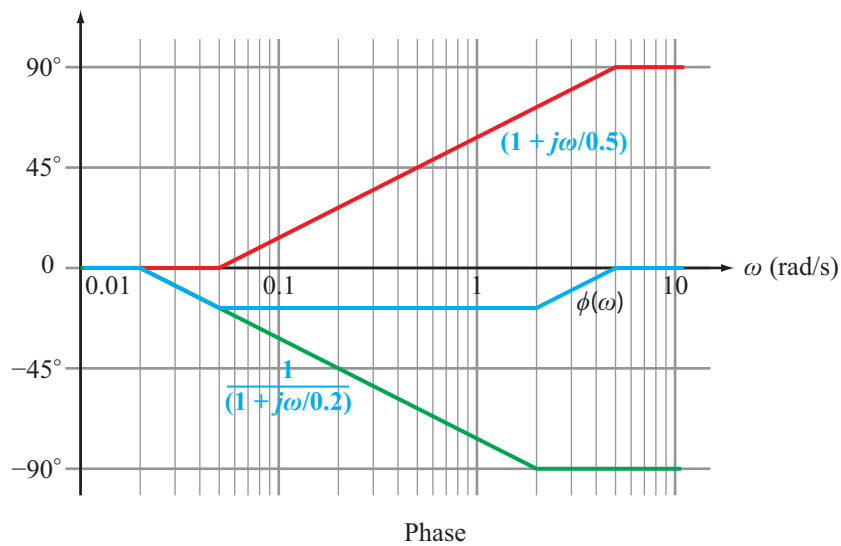
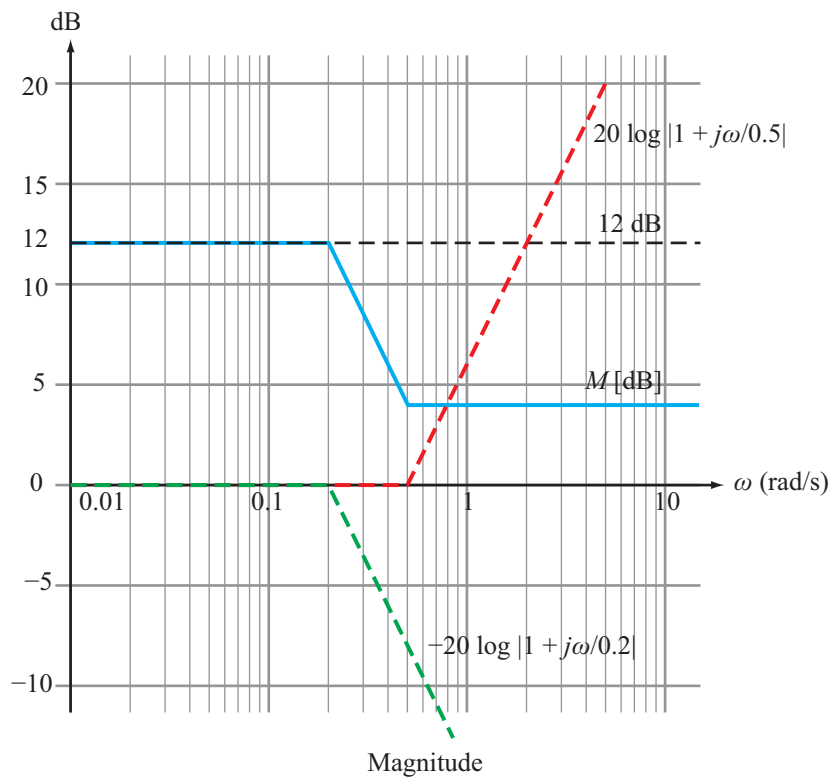
- Line starts at 60 dB at $\omega = 1$ rad/s, and has slope of -40 dB/decade
 - Constant factor 1000 \implies 60 dB
 - Pole @ origin of order 2
- Simple zero with $\omega_c = 50$ rad/s, of order 2



$$\text{(c)} \quad \mathbf{H}(\omega) = \frac{40 + j80\omega}{10 + j50\omega} = \frac{40(1 + j2\omega)}{10(1 + j5\omega)} = \frac{4(1 + j\omega/0.5)}{(1 + j\omega/0.2)}.$$

- Constant factor 4 \implies 12 dB
- Simple pole with $\omega_c = 0.2$ rad/s
- Simple zero with $\omega_c = 0.5$ rad/s

$$M [\text{dB}] = 20 \log |\mathbf{H}(\omega)| = 20 \log 4 + 20 \log |1 + j\omega/0.5| - 20 \log |1 + j\omega/0.2|$$



(d)

$$\begin{aligned} \mathbf{H}(\omega) &= \frac{(20 + j5\omega)(20 + j\omega)}{j\omega} \\ &= \frac{-j20(1 + j\omega/4)20(1 + j\omega/20)}{\omega} = \frac{-j400(1 + j\omega/4)(1 + j\omega/20)}{\omega} \end{aligned}$$

- Constant term 400 \Rightarrow 52 dB
- Pole @ origin

- Simple zero with $\omega_c = 4$ rad/s
- Simple zero with $\omega_c = 20$ rad/s

