1. Speaker System

Your job is to construct a speaker system that operates in the range from 20 Hz to 20,000 Hz. Such a system would ideally have a transfer function $H_{\text{desired}}(f)$, where the magnitude plot of $H_{\text{desired}}(f)$ is as follows:

Within the operating range from 20 Hz to 20,000 Hz, the transfer function has magnitude equal to 1, and otherwise it is $10^{-3}$.

$$|H_{\text{desired}}(f)| = \begin{cases} 1 & \text{if } f \in [20, 20000], \\ 10^{-3} & \text{else.} \end{cases}$$

Say you already have the parts to make a speaker system that operates in the midrange section of the audio frequency spectrum, $f$ between 250 Hz to 4,000 Hz. Mathematically, this speaker has a transfer function $H_1$ where

$$|H_1(f)| = \begin{cases} 1 & \text{if } f \in [250, 4000], \\ 10^{-3} & \text{else.} \end{cases}$$

(a) **Draw the bode plot of $|H_1(f)|$.**

(b) You want to build upon what you currently have to build the desired speaker system. You already have a system that implements the transfer function $H_1(f)$. Your system for $H_1(f)$ can be cascaded with another system in series. In between these two systems, you place a unity gain buffer to decouple these
systems. In other words, your second system can be composed with your first system. This means that the resulting transfer function of the composition is the multiplication of the two systems.

\[ H_{\text{composition}}(f) = H_1(f) \cdot H_2(f), \]

where \( H_2(f) \) is the transfer function of the second system.

The goal of the composition is to achieve \( H_{\text{desired}}(f) \). Draw the Bode plot of \(|H_2(f)|\) such that \(|H_{\text{composition}}(f)| = |H_{\text{desired}}(f)|\).

2. Phun with Phasors

(a) Suppose that we have a voltage source \( v(t) = 4 \sin(t) \) V connected in parallel to a resistor with \( R = 2\Omega \). Draw this circuit.

(b) For the previous part, compute the phasor \( \tilde{V} \) that represents the voltage across the voltage source. Draw this phasor \( \tilde{V} \) as a “vector” on the complex plane. (HINT: Remember that the phasor is the coefficient of the complex exponential corresponding to the positive frequency.)

(c) Continuing, what is the phasor \( \tilde{I}_R \) for \( i_R(t) \), the current into the resistor? Draw this phasor \( \tilde{I}_R \) on the complex plane.

(d) Suppose that instead of a resistor, we had an inductor with inductance \( L = 4 \) H. What is the phasor \( \tilde{I}_L \) for \( i_L(t) \), the current into the inductor? Draw this phasor \( \tilde{I}_L \) on the complex plane.

(e) Suppose that instead of an inductor or resistor, we have a capacitor \( C = \frac{1}{2} \) F connected in parallel with this voltage source. What is the phasor \( \tilde{I}_C \) for \( i_C(t) \), the current into the capacitor? Draw this phasor \( \tilde{I}_C \) on the complex plane.

(f) Derive the expression for the impedance \( Z_L \) in terms of the angular frequency \( \omega \) and the inductance \( L \) from first principles. You can assume:

- The definition of the phasor representation is the coefficient of \( e^{j\omega t} \) in any real sinusoidal quantity at frequency \( \omega \).
- The definition of impedance as the ratio of the phasor representation of the voltage across an element to the phasor representation of the current through the element.
- The basic differential equation \( V_L(t) = L \frac{d}{dt} I_L(t) \) that governs an inductor’s current-voltage relationship.

(g) If the time-domain current waveforms \( i_1(t), i_2(t), i_3(t) \) entering a node satisfy KCL (i.e. sum to zero) at that node for all time, show that their phasor representations \( \tilde{I}_1, \tilde{I}_2, \tilde{I}_3 \) also must satisfy KCL at that node.

3. Analog-to-Digital Converter (ADC)

In this question, we’ll be examining common pitfalls of driving successive approximation ADCs, which are the kind of ADCs found on many devices, including the MSP430s you use in lab.

We can model the ADC internally as having a switch connected to a sampling capacitor with capacitance \( C \). In order to read an analog voltage, the ADC closes the switch for a sampling period \( \tau \). Charge collects on the capacitor during this time. At the end of the sampling period, the switch is opened and the voltage across the capacitor \( V_c \) is read and converted into a digital value.
(a) Our ADC can only read voltages from 0V to 3V. We have a voltage source \( V_s \) we want to measure that we know outputs from 0V to 9V. Your lab partner suggests you use a resistor divider with resistors \( R_1 \) and \( R_2 \) so you can read the entire range of the voltage source.

**Find the equation for the voltage on the capacitor \( V_c(t) \) for \( 0 \leq t \leq \tau \) (when the switch is closed) assuming \( V_c(0) = 0 \).**

\[
\frac{V_s}{R_1} + \frac{V_s}{R_2} = V_c(t)
\]

![ADC circuit charged via voltage divider](image1)

(b) The nominal voltage we want to read is \( V_n = \frac{R_2}{R_2 + R_1} V_s \). Suppose \( R_2 = 100k\Omega, \ R_1 = 200k\Omega \), and \( C = 10pF \). **What is the minimum sampling time \( T_{\text{min}} \) such that our measured \( V_c(T_{\text{min}}) \) is within 5% of \( V_n \)? (That is, \( V_{\text{nom}} - V_c(T_{\text{min}}) \leq 0.05 V_{\text{nom}} \).)**

(Note: Some useful approximations are \( e^{-1} = 0.37 \), \( e^{-2} = 0.14 \), \( e^{-3} = 0.05 \), \( e^{-4} = 0.02 \).)

(c) Your lab partner thinks this time is too slow and that “we gotta go faster” to be able to sample inputs that are quickly changing. Using a single op-amp, how could you modify the non-ADC part of the circuit to look like an ideal voltage source with output voltage \( V_n \)?

(d) Now that the voltage input to the ADC is ideal, we need to consider the effects of the parasitic inductance \( L \) and resistance \( R_3 \) in the circuit. **Set up a system of differential equations for \( V_c(t) \) and \( i(t) \) in matrix form and solve for the eigenvalues of the system.**

\[
\begin{align*}
\frac{V_c}{C} & = i(t) \\
\frac{L}{C} & = \frac{V_c}{R_3} + \frac{1}{R_3} \cdot i(t)
\end{align*}
\]

![ADC circuit with perfect voltage source inductor, capacitor and resistor](image2)

(e) \( C = 10pF \) and \( L = 1nH \). We can increase \( R_3 \) by adding series resistance to our voltage source. **What is the minimum value \( R_3 \) should be to eliminate all oscillation in \( V_c(t) \)?**

4. DC Motor Driver

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In this question, we’ll examine the operation of a DC motor driver similar to the one you’ll make in lab.

Our circuit model for a motor is a resistor with resistance $R_{motor}$ and inductor with inductance $L_{motor}$. Our motor has an ideal diode in parallel with the inductor. Remember that an ideal diode is an open circuit when there is negative voltage across it, but is a closed circuit when there is positive voltage across it.

(a) To begin with, we’ll model the supply as an ideal voltage source $V_{sup}$. At time $t = 0$ we close the switch to provide current to the motor. If we leave the switch closed long enough that the circuit is in steady state, what will the current $I_{max}$ through the motor resistor $R_{motor}$ be? This is the maximum possible current through the motor.

(b) Let’s now assume that we open the switch again before the circuit reaches steady state, because we don’t want the current to actually reach this maximum value.

We want to control the torque of the motor, which means we want $I_{motor}(t)$ to have a time-averaged value of $I_{avg} = \frac{I_{max}}{4}$. Assuming $I_{motor}(0) = I_{avg}$ and that $V_{sup} > V_{fwd}$, at what time $t = T_1$ should we open the switch so that $I_{motor}(T_1) = 1.1 \cdot I_{avg}$? Answer symbolically, without plugging in for particular component values.

(c) Now that $I_{motor} = 1.1 \cdot I_{avg}$ we’ll open the switch and reset our timer so $t = 0$. In this new time frame, $I_{motor}(0) = 1.1 \cdot I_{avg}$. At what time $T_2$ will $I_{motor}(T_2) = I_{avg}$? Leave your answer symbolically.

(d) Based on the previous sections, we will control the average current through the motor by alternating between opening the switch for $T_1$ time and closing the switch after $T_2$ time. We can now model the motor and switch as a current sink attached to our supply that alternates between sinking $I_{avg}$ current for $T_1$ time (when the switch is open) and sinking 0 current for $T_2$ time (when the switch is closed), seen in the circuit below.

Assume for now that $C = 0F$. Sketch a graph for $V_{sup}(t)$ for from time $0 \leq t \leq T_1 + T_2$. Make sure to label the value for $V_{sup}(t)$ at time $t = 0$, $t = T_1$, and $t = T_1 + T_2$. Let $I_{sup}(t)$ in Figure 6 be the $I_{sup}(t)$
in the waveform shown below.

Figure 5: Current sink over time

Figure 6: Complete motor driver circuit

(e) Given $V_s = 9\text{V}$, $R = 3\Omega$, $I_{\text{avg}} = 3\text{A}$, and $V_{\text{sup}}(0) = V_s$, for what value of $C$ will $V_{\text{sup}}(T_1) = 0.9 \cdot V_s$? You may leave $T_1$ as a variable. (Note: You may use the approximation $e^{-0.1} \approx 0.9$)

5. High-pass Filter

You have a $1\text{k}\Omega$ resistor and a $1\text{mH}$ inductor wired up as a high-pass filter.

(a) **Draw the filter, labeling the input node, output node, and ground.**

(b) **Write down the transfer function of the filter, $H(\omega)$.**

(c) **Draw a straight-line approximation to the Bode plot (both magnitude and phase) of the filter on the attached graph paper.** Label the horizontal and vertical axes clearly.
(d) **Annotate your Bode plot with three dots, each representing where the straight line approximation has its worst errors.** One dot should be on the magnitude plot, and two should be on the phase plot. Label each dot with the error at that point.

(e) **Write down the steady state response of this filter to an input voltage** $V(t) = -10\cos\left(10^6 t + \frac{\pi}{3}\right)$ V.

6. **RLC Circuit: State-Space representation**

Consider the RLC circuit you saw in HW2.
(a) Let \( x_1(t) = V_C(t) \) be the voltage on the capacitor, and let \( x_2(t) = i_L(t) \) be the current through the inductor. Let \( \bar{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \). Write out the system of differential equations governing this circuit to express it as:

\[
\frac{d}{dt} \bar{x}(t) = A\bar{x}(t) + \bar{b}V_S(t)
\]  \hspace{1cm} (1)

(b) What are the eigenvalues of \( A \) in terms of \( R, L, C \)?

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