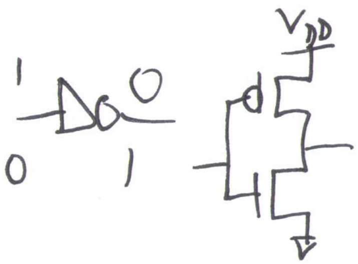
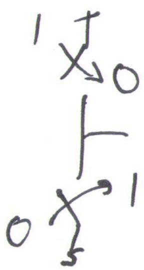
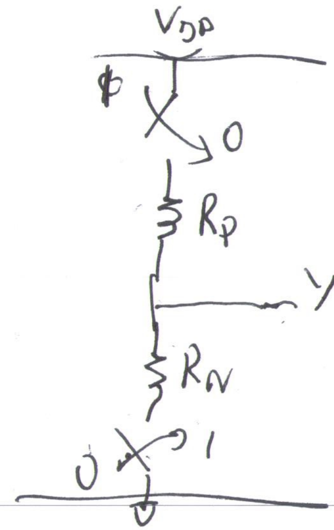
OH Tuesday 12:30-1:30 144MA Cory

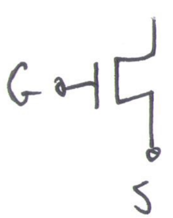
HW parties

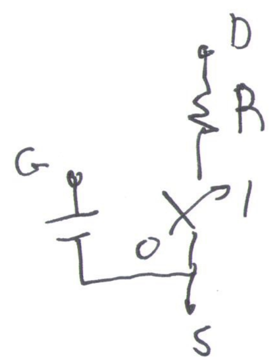
my office 2^{9} Cory

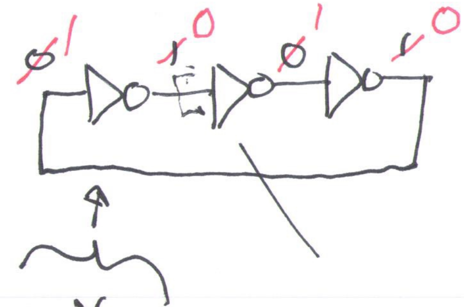




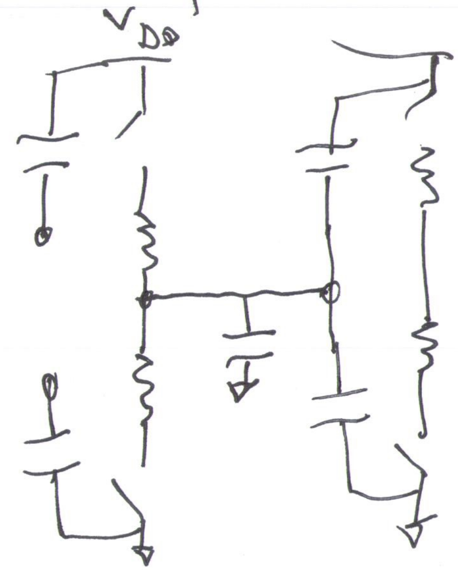


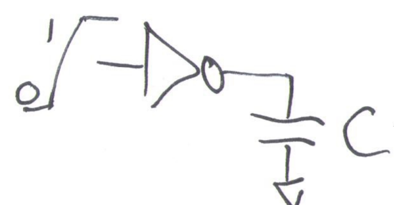


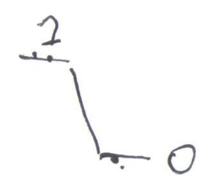




Ring Oscillator







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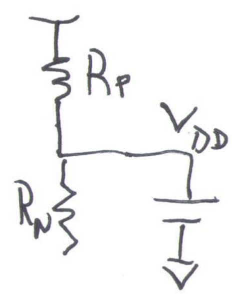

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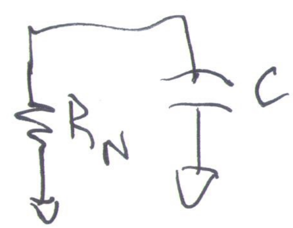

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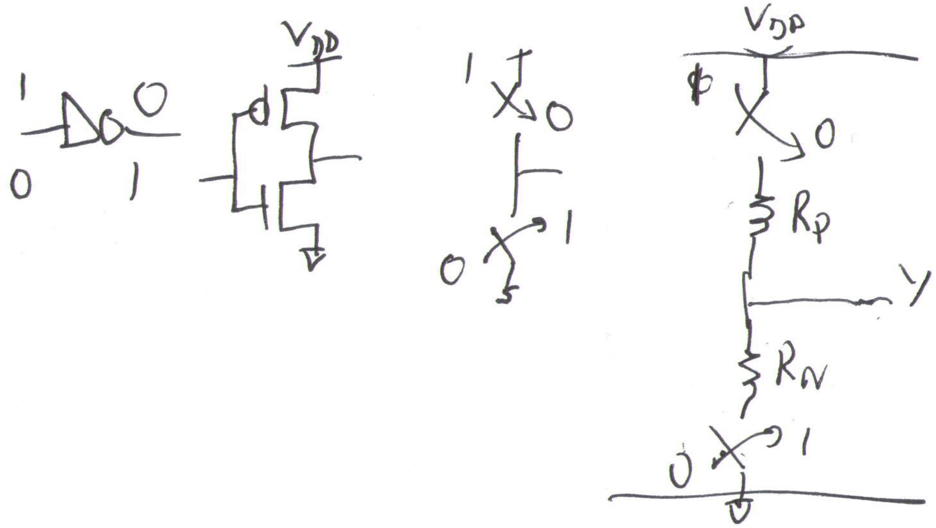
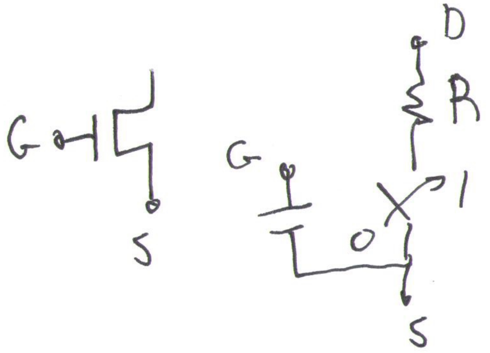
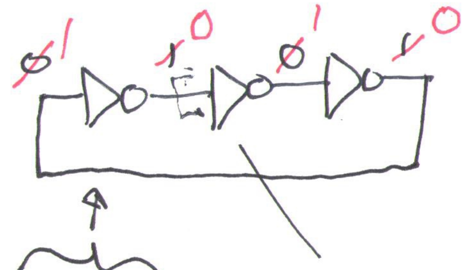
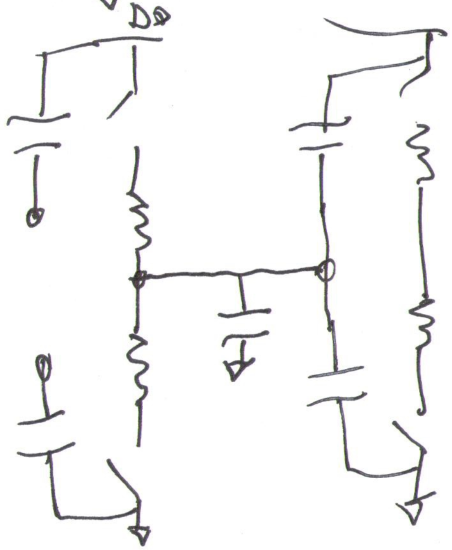
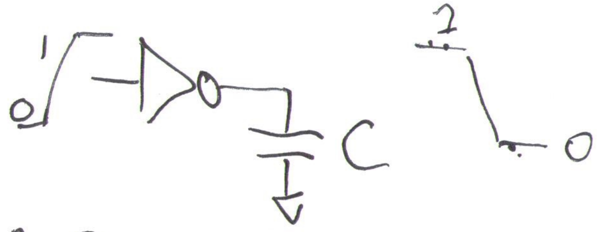






Ring Oscillator





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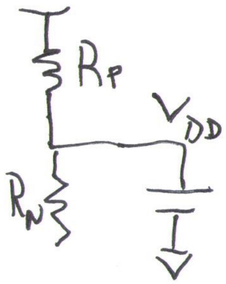

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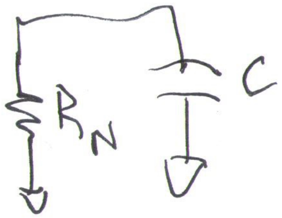
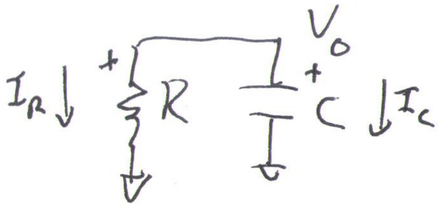
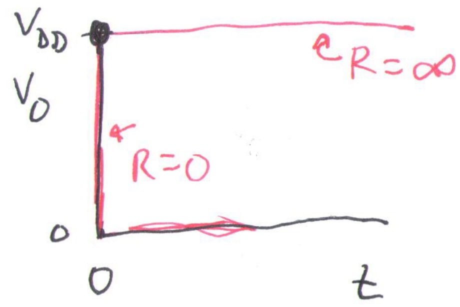






$V_o(t=0) = V_{DD}$

$Q = CV$

$\frac{d}{dt}Q = I_c = \frac{d}{dt}(CV) = C\frac{dV}{dt}$

$U = \frac{1}{2}CV^2$

$I_c = C\frac{dV_o}{dt} = -I_R = \frac{-V_o}{R}$

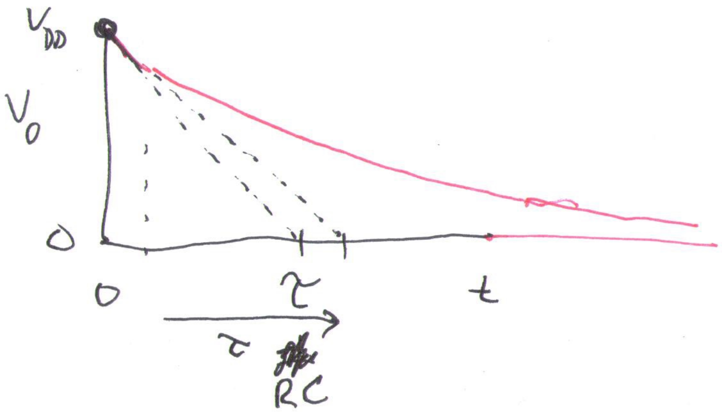
$I_R = \frac{V_o}{R}$

$I_c = -I_R$

$\frac{dV_o}{dt} = \frac{-V_o}{RC}$ 1st order Ordinary Diff. Eqn.

$\tau = RC$ time constant

$\frac{dV_o}{dt} = \frac{-V_o}{\tau}$



$\frac{dV_o}{dt} = \frac{-V_o}{\tau}$

$\frac{d}{dt}[V_o] = \lambda [V_o]$

differential operator

eigenvalues

eigenfunctions

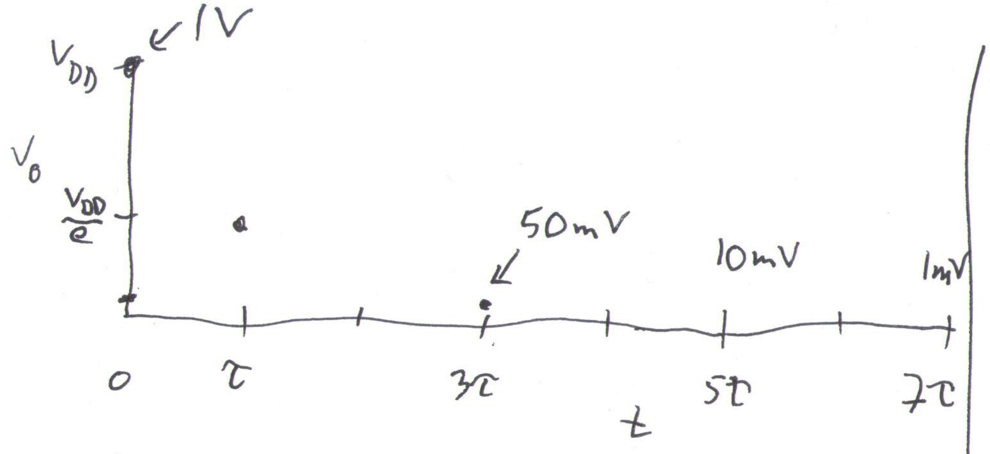
$V_o = Ke^{-t/\tau}$

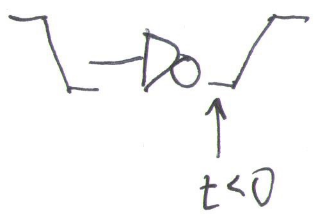
$V_o = Ke^{at}$

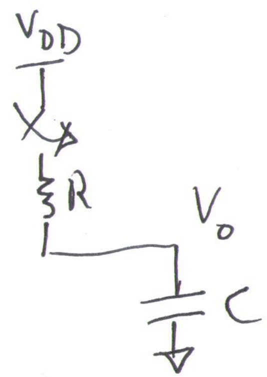
$V_o(t=0) = V_{DD} = Ke^{-0/\tau} = K$

$V_o = V_{DD}e^{at}$

$V_o(t) = Ke^{-t/\tau}$

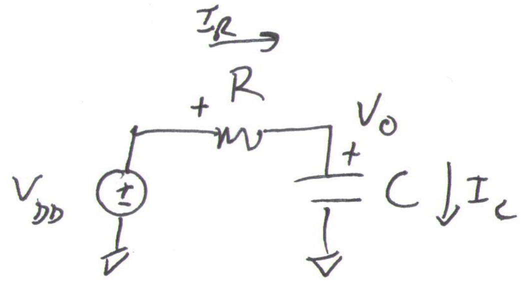






$V_{DD}\, e^{-t/\tau}$

t	$e^{-t/\tau}$
τ	0.37
3τ	0.05
5τ	0.01
7τ	0.001



$$I_c = C\frac{dV_o}{dt} \qquad C\frac{dV_o}{dt} = \frac{V_{DD}-V_o}{R}$$

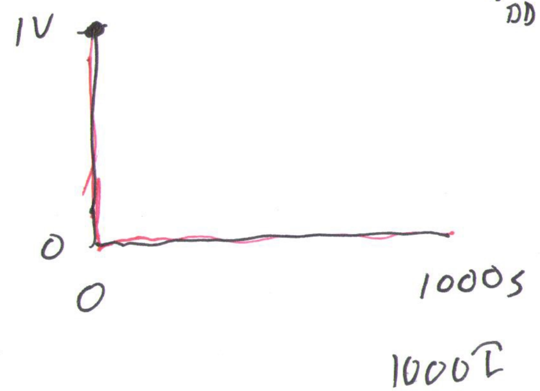
$$I_R = \frac{V_{DD}-V_o}{R}$$

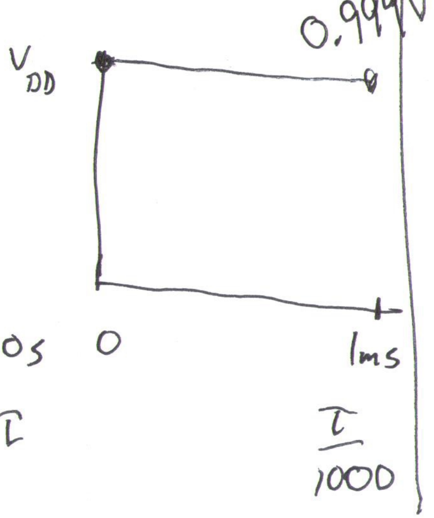
$$\frac{dV_o}{dt} = \frac{-V_o}{RC} + \frac{V_{DD}}{RC}$$

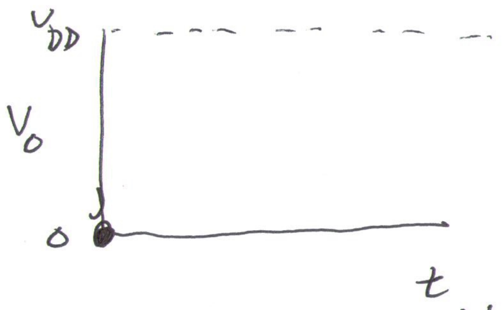
$$I_c = I_R$$

EX; $R = 1\Omega$ $\quad C = 1F$ $\quad V_{DD} = 1V$

$\tau = RC = 1s$







$V_o(t=0) = 0$

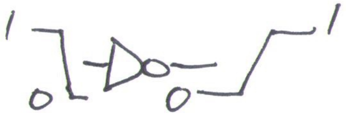
$$V_o(t) = V_{final} + V_{transient}$$

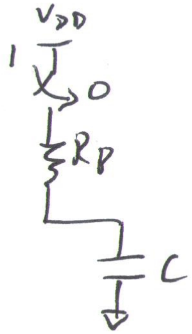
$$= V_{DD} + V_{transient}(t)$$

$$V_o(0) = 0 = V_{DD} + V_{transient}(0)$$

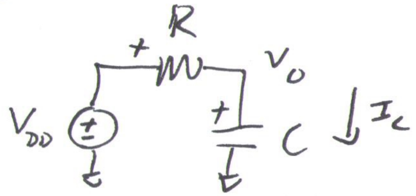
$$V_{transient}(0) = -V_{DD}$$

what about pull-up?





$V_o(t=0)=0$



$$I_c = C\frac{dV_o}{dt}$$

$$I_R = \frac{V_{DD}-V_o}{R} = I_c = C\frac{dV_o}{dt}$$

$$\frac{dV_o}{dt} = \frac{-V_o}{RC} + \frac{V_{DD}}{RC}$$ non-homogeneous

Assume $V_o = V_{final} + V_{transient} = V_{DD} + V_{transient}$

V_{final}: const, $= V_{DD}$

$$\frac{dV_o}{dt} = \frac{d}{dt}V_{transient}$$

$$\frac{d}{dt}V_{transient} = \frac{-(V_{DD}+V_{transient})}{RC} + \frac{V_{DD}}{RC}$$

$$= \frac{-V_{transient}}{RC}$$

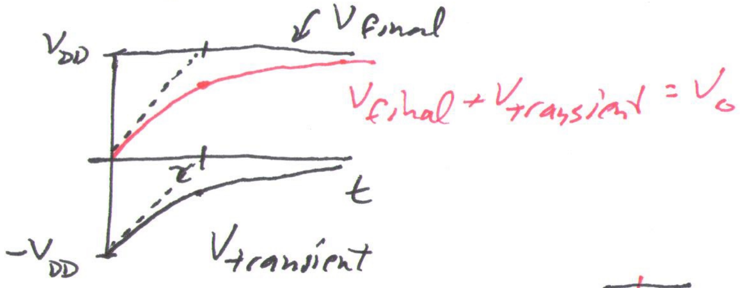
$$V_{transient}(t) = Ke^{-t/RC}$$

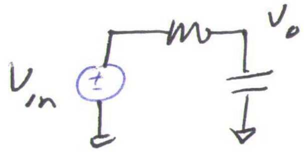
$$V_{transient}(0) = -V_{DD} = Ke^{0}$$

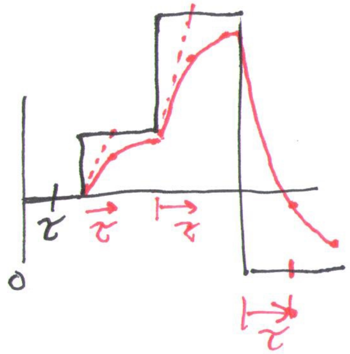
$$V_{transient} = -V_{DD}e^{-t/RC}$$

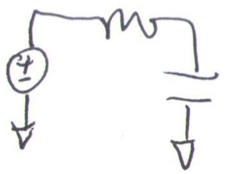
$$V_o = V_{DD} + V_{transient} = V_{DD} - V_{DD}e^{-t/RC}$$

$$= V_{DD}\left(1-e^{-t/RC}\right)$$









$$\frac{dV_o}{dt} = \frac{-V_o}{RC} + \frac{V_{DD}}{RC}$$ circuit analysis

$$V_o(t) = V_{DD} + V_{transient}(t)$$

$$V_{transient}(0) = -V_{DD}$$

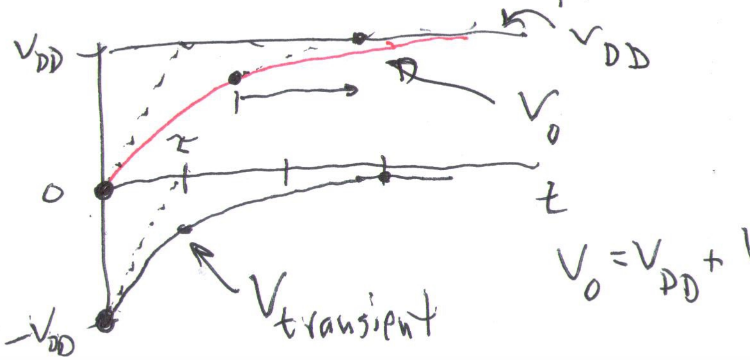
$$\frac{dV_o}{dt} = \cancel{\frac{dV_{DD}}{dt}}^{\,0} + \frac{dV_{transient}}{dt}$$

$$\frac{dV_{transient}}{dt} = \frac{-(V_{DD} + V_{transient})}{RC} + \frac{V_{DD}}{RC}$$

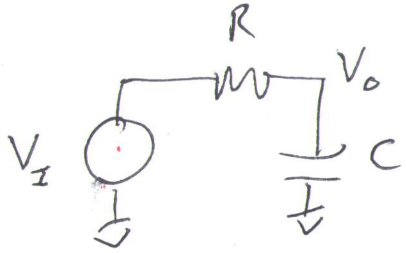
$$\frac{dV_{transient}}{dt} = \frac{-V_{transient}}{RC}$$

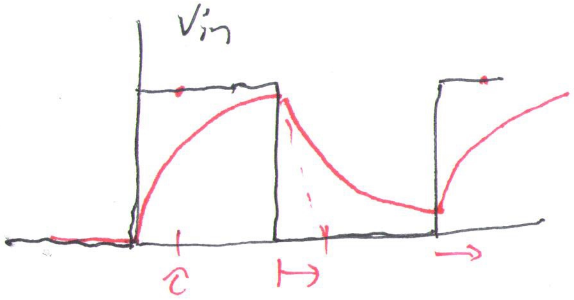
$$V_{transient} = K e^{at} \qquad a = \frac{-1}{RC}$$

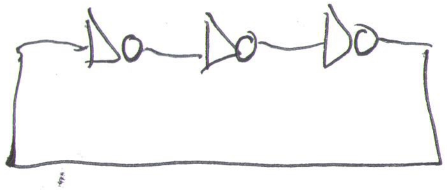
$$K = -V_{DD}$$



$$V_o = V_{DD} + V_{transient}$$







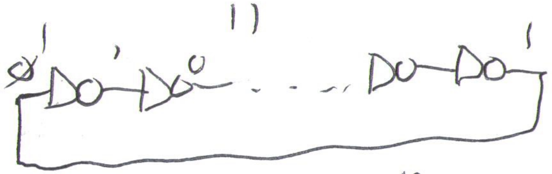
1970 $L = 20\,\mu m$

2019 $L \simeq 10\,nm$

$\tau \downarrow$

EX: 11 stage rig osc.

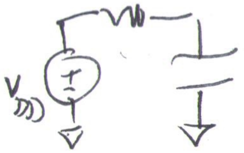
32 nm CMOS



$$f_{osc} = 10\,GHz = 10^{10}\,\frac{cycles}{sec}$$

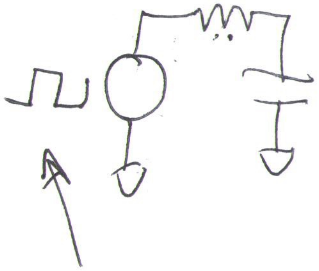
Capacitance/inverter $\simeq 1\,fF$

total cap $= C_{?} = 10\,fF$



$$U_C = \frac{1}{2} C V_{DD}^2$$

$$Q_C = C V_{DD}$$



$$\Delta E_{supply} = -Q_C V_{DD}$$

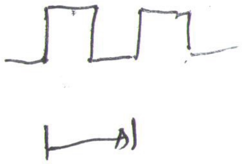
$$= -C V_{DD}^2$$

f $\frac{\text{cycles}}{\text{sec}}$

$$\boxed{P_{dissipated} = C V_{DD}^2 f}$$

$L = 32\,nm$

Ex:



$f = 10^{10} \frac{cyc}{s}$

$C \approx 10\,fF$

$V \approx 1V$

$$P = C V_{DD}^2 f$$

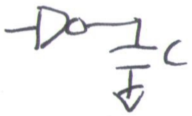
$$= (10^{-14} F)(1V)^2 \, 10^{10} \tfrac{1}{s}$$

$$= 10^{-4} W = 100\,\mu W$$

$$\text{Area} = 2\,\mu m^2$$

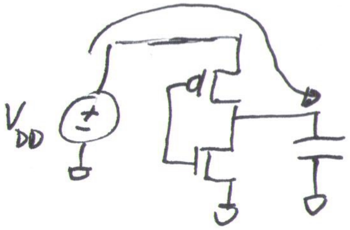
$$= 2 \times 10^{-6}\,mm^2$$

switching power



$U_C = \frac{1}{2} C V_{DD}^2$, or 0

$Q_C = C V_{DD}$, or 0



$$\Delta E_{supply} = \Delta (QV)_{supply}$$
$$= \Delta Q \, V_{DD}$$
$$= -Q_C \, V_{DD}$$
$$= -C V_{DD} \, V_{DD}$$
$$= -C V_{DD}^2$$

Supply loses $C V_{DD}^2$ energy each time cap is charged.

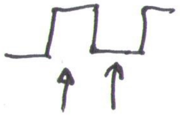
Cap gains $\frac{1}{2} C V_{DD}^2$ " " " "

Resistor dissipates $\frac{1}{2} C V_{DD}^2$ " " "

when switch f times per second, power is consumed

$$\boxed{P = C V_{DD}^2 f}$$

EX: 32nm 11 stage CMOS ring osc.



everyone flips once | everyone flips again

$V_{DD} = 1V$

$C_{total} = 11 (C_{inv})$

$= 10 fF$

$f = 10 GHz$ measured

$P = C_{total} V_{DD}^2 f$

$= (10^{-14} F)(1V)^2 10^{10}$

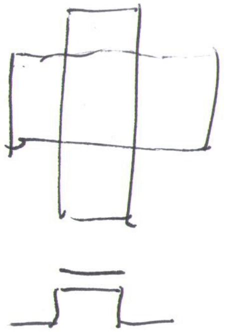
$= 10^{-4} W$ ← measured

area $\simeq 2 \mu m^2 = 2 \times 10^{-6} mm^2$

$$\frac{P}{A} = \frac{10^{-4} W}{2 \times 10^{-6} mm^2} = \frac{1}{2} 10^2 \frac{W}{mm^2}$$
$$= 50 \frac{W}{mm^2}$$

$> 2 \times$ heat flux at surface of sun

Moore's law, transistor scaling



features shrink

$20\,\mu m \rightarrow 20\,nm$

capacitance decreases ~

resistance decreases

time constant (RC) decreases

speed goes up

power per gate goes down

power per <u>area</u> goes <u>up</u>

around $100 \frac{W}{cm^2}$, cooling is primary problem

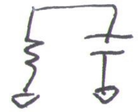
can make 10 GHz processors. Can't cool them!

cell phone: 6 W peak (3 GHz?)

for $\approx$ 10 ms

average $\approx$ 0.8 W

Power in resistor



$$V_o = V_{DD} e^{-t/RC}$$

$$I_c = C \frac{dV_o}{dt} = -\frac{CV_{DD}}{RC} e^{-t/RC}$$

$$= \frac{-1}{R} V_{DD} e^{-t/RC}$$

$$I_R = \frac{V_o}{R} = \frac{V_{DD}}{R} e^{-t/RC}$$

$$= -I_c \quad \text{check } \checkmark$$

$$P_R = IV = \left(\frac{V_{DD}}{R} e^{-t/RC}\right)\left(V_{DD} e^{-t/RC}\right)$$

$$= \frac{V_{DD}^2}{R} e^{-2t/RC}$$

$$E_R = \int_0^\infty P_R \, dt = \int_0^\infty \frac{V_{DD}^2}{R} e^{-2t/RC} dt$$

$$= \frac{1}{\left(\frac{-2}{RC}\right)} \frac{V_{DD}^2}{R} e^{-2t/RC} \Big|_0^\infty$$

$$= \frac{-1}{2} C V_{DD}^2 e^{-2t/RC} \Big|_0^\infty$$

$$= -\left(\frac{1}{2} C V_{DD}^2\right)\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{2} C V_{DD}^2$$

all cap energy dissipated in resistor $\checkmark$