1 Controller Canonical Form - Introduction

a) Show that a discrete-time system in controllable canonical form is essentially a higher order scalar recurrence relation with scalar input.

b) Show that a continuous-time system in controllable canonical form is essentially a higher order scalar differential equation with scalar input.

2 Controller Canonical Form - Eigenvalues Placement

Consider the following linear discrete time system

\[
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -3 & -4
\end{bmatrix}
\begin{bmatrix}
x(t+1) \\
x(t) \\
u(t)
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

\[
\begin{bmatrix}
x(t+1) \\
x(t) \\
u(t)
\end{bmatrix}
\begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -3 & -4
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]
a) Is this system controllable?

b) Is the linear discrete time system stable?

c) Using state feedback $u(t) = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \hat{x}(t)$ place the eigenvalues at $0, 1/2, -1/2$. 