1 RC Circuits

In this problem, we will be using differential equations to find the voltage across a capacitor over time in an RC circuit. We set up our problem by first defining three functions over time: $I(t)$ is the current at time $t$, $V(t)$ is the voltage across the circuit at time $t$, and $V_c(t)$ is the voltage across the capacitor at time $t$.

Recall from 16A that the voltage across a resistor is defined as $V_R = RI_R$ where $I_R$ is the current across the resistor. Also, recall that the voltage across a capacitor is defined as $V_C = \frac{Q}{C}$ where $Q$ is the charge across the capacitor.

![Example Circuit](image1)

Figure 1: Example Circuit

1. First, find an equation that relates the current across the capacitor $I(t)$ with the voltage across the capacitor $V_C(t)$.

2. Write a system of equations that relates the functions $I(t)$, $V_C(t)$, and $V(t)$.

3. Rewrite the previous equation in part (b) in the form of a differential equation involving only $V_C(t)$ and $V(t)$.

![Circuit for part (d)](image2)

Figure 2: Circuit for part (d)

4. Let’s suppose that at $t = 0$, the capacitor is charged to a voltage $V_{DD}$ ($V_C(0) = V_{DD}$). Let’s also assume that $V(t) = 0$ for all $t \geq 0$. Solve the differential equation for $V_C(t)$ for $t \geq 0$. 
5. Now, let’s suppose that we start with an uncharged capacitor $V_C(0) = 0$. We apply some constant voltage $V(t) = V_{DD}$ across the circuit. Solve the differential equation for $V_C(t)$ for $t \geq 0$.

2 Systems of Differential Equations

Consider the following circuit.

1. Write a system of differential equations that governs the voltages $V_{C1}, V_{C2}$ across the capacitors. Use the following values: $C_1 = 1 \mu F$, $C_2 = \frac{1}{3} \mu F$, $R_2 = \frac{1}{2} M \Omega$, $R_1 = \frac{1}{3} M \Omega$.

2. Suppose also that $V_{in}$ was at 7V for a long time, and then transitioned to be 0V at time $t = 0$. Write the system of differential equations that are valid for $t \geq 0$ in matrix form. What are the initial conditions?

3. Find the eigenvalues $\lambda_1$, $\lambda_2$ and eigenspaces for the matrix corresponding to the differential equation matrix above.