

Consider a first order continuous time linear system with dynamics given by:

$$\dot{x} = -x + u \quad (1)$$

In discrete time, a “zero-order-hold” holds the value of the control input $u(t)$ constant for one time step T , i.e. the control $u(t) = u(kT)$ for $kT \leq t < kT + T$.

Let the error $e(t) = r(t) - x(t)$, and a feedback controller is chosen such that $u(t) = 3(x(t) - r(t))$ where $r(t)$ is the reference. Here $r(t)$ is the unit step.

Assume the system has initial condition $x(0) = 0$. The state and error are listed in the table below at each T . Note that the system is continuous, but the control value is held constant and updated at each T .

t (sec)	x(t)	e(t) = r(t) - x(t)	u(t)
0 ⁻	0	0	0
0	0	1	3
1	2	-1	-3
2	-1	2	6
3	3.5	-2.5	-7.5
4	-3.5	4.5	13.5
5	6.5	-5.5	-16.5

This example is unstable. By reducing the sampling period, (even while keeping the gain the same), the control system can be made stable.

