Fig. 1 Simplified motor model without inductance.

Given: \( R_m = 2\Omega \), \( k_r = 0.1 Nm^{-1} \), \( k_e = 0.01V - s/rad \)

As derived in class, the motor torque depends on motor current: \( \tau = k_r i_m \), where \( \tau \) has units of \( N - m \), and \( k_r \) is \( N - m/amp \).

The back emf voltage is proportional to motor velocity: \( V_e = k_e \dot{\theta}_m \), where \( \dot{\theta}_m \) is motor velocity in radians/second and thus \( k_e \) has units of \( Volt - sec/rad \).

For the physics behind the motor model, see:

Problems:

1. The unloaded motor is connected to a 6V battery. Neglecting friction and other losses, determine \( i_m, V_e, \) and \( \dot{\theta}_m \) in steady state.

2. The motor is connected to a 6V battery with negligible internal battery resistance. The motor shaft is clamped so that \( \theta_m = 0 \). Determine \( i_m, V_e, \) and \( \tau_m \).

3. The motor is connected through a gear box to a car tire. The motor is turning at 5000 rpm. What is the instantaneous open circuit voltage \( V_m \)?

4. The motor is turning at 5000 rpm, and \( V_m \) is now short circuited. What is the initial current \( i_m \), and torque \( \tau_m \) shortly after the short circuit is applied?

For the following, consider that the motor is connected through a gear box to a car tire. The motor is initially turning at 5000 rpm, and the car has inertia and friction.

5. Consider \( V_m \) is short circuited at \( t = 0 \). Sketch the trend of \( i_m, V_e, \dot{\theta}_m, \) and \( \tau_m \) until the car comes to a rest.

6. Consider now \( V_m \) connected to battery at -1ms and short circuited for 2 ms at \( t=0 \). Sketch variables as above for -1 ms \(< t < 2 \) ms.

7. Consider motor initially disconnected. Now \( V_m \) is connected to a 6V battery with negligible internal resistance at \( t = 0 \) ms. Sketch the trend of \( i_m, V_e, \dot{\theta}_m, \) and \( \tau_m \) until the car reaches a steady state velocity.

8. Consider now \( V_m \) is connected to the 6V battery for 1 ms at \( t=0 \), and then \( V_m \) is open circuited. Sketch variables as above for -1 ms \(< t < 2 \) ms.