

Consider a first order continuous time linear system with dynamics given by:

$$\dot{x} = -x + u \quad (1)$$

In discrete time, a “zero-order-hold” holds the value of the control input  $u(t)$  constant for one time step  $T$ , i.e. the control  $u(t) = u(kT)$  for  $kT \leq t < kT + T$ .

Let the error  $e(t) = r(t) - x(t)$ , and a feedback controller is chosen such that  $u(t) = 3(x(t) - r(t))$  where  $r(t)$  is the reference. Here  $r(t)$  is the unit step.

Assume the system has initial condition  $x(0) = 0$ . The state and error are listed in the table below at each  $T$ . Note that the system is continuous, but the control value is held constant and updated at each  $T$ .

t (sec)	x(t)	e(t) = r(t) - x(t)	u(t)
0 <sup>-</sup>	0	0	0
0	0	1	3
1	2	-1	-3
2	-1	2	6
3	3.5	-2.5	-7.5
4	-3.5	4.5	13.5
5	6.5	-5.5	-16.5

This example is unstable. By reducing the sampling period, (even while keeping the gain the same), the control system can be made stable.

