5.1) **Retarded Potential (time-domain):** Consider the scalar potential produced by two time-varying sources. One source is at \( x_1 = (0,0,0) \) and has charge \( q_1(t) \), the other is at \( x_2 = (a,0,0) \) and has charge \( q_2(t) \).

a) Using the scalar potential from a point charge write down a general expression for \( \Phi(x,y,z,t) \).

b) Find the Electric field contributed by this potential \( E(x,y,z,t) \) far from the source in the \( z = 0 \) plane.

c) Repeat part b) for a large distance in a direction \( \mathbf{n} = (\cos \phi, \sin \phi, 0) \).

d) Now suppose \( q_2(t) = q_1(t - \tau) \). Find the value of \( \tau \) that will synchronize the contributions from the two sources regardless of the time-variation of \( q_1(t) \).

e) Show how the delay can be described through using \( (x_2 \cdot n) \).

f) Find \( E(R,0,0,t) E(R,0,0,t) \) where \( R \gg a \), and show that it is given by \( \mathbf{E}_1 \mathbf{E}_1 + \mathbf{E}_2 \mathbf{E}_2 \) plus a cross term proportional to \( \mathbf{E}_1 \mathbf{E}_2 \) and involving the delay between the sources.

5.2) **Retarded Potential (frequency-domain):** Now reconsider the two point time-varying source in Problem 5.1.

a) Find an expression for \( \Phi(x,y,z,\omega) \).

b) Find the Electric field contributed by this potential \( E(x,y,z,\omega) \) far from the source in the \( z = 0 \) plane. Hint use \( n = (\cos \phi, \sin \phi, 0) \).

c) Using the curl \( \mathbf{E} \) Maxwell equation find \( \mathbf{H}(x,y,z,\omega) \) far from the source in the \( z = 0 \) plane.

d) Show that \( \mathbf{H}(x,y,z,-\omega) = \mathbf{H}^*(x,y,z,\omega) \).

e) For the case of \( q_2(t) = q_1(t - \tau) \), find \( q_2(\omega) \) in terms of \( q_1(\omega) \).

f) Find the product \( E(R,0,0,\omega) E(R,0,0,\omega) \) where \( R \gg a \), and show that it is given by \( \mathbf{E}_1 \mathbf{E}_1 + \mathbf{E}_2 \mathbf{E}_2 \) plus a cross term proportional to \( \mathbf{E}_1 \mathbf{E}_2 \) and involving the phase between the sources and \( (x_2 \cdot n) \).

4.3) **Green’s Function in Time-harmonic:** Consider the interior of a grounded box defined by the six planes, \( x = 0, y = 0, z = 0, x = a, y = b, \) and \( z = c \). A time-varying charge source is given by \( q(x,y,z,t) = \delta(x-d)\delta(y-e)\delta(z-f)\delta(t-\tau) \).

a) Convert this source to a Fourier representation using \( q(x,y,z,\omega) \).

b) Use the N-dimensional eigenfunctions and the scaler wave equation to find the solution for the potential inside the box.

c) Describe what happens to the potential when the time-harmonic frequency contribution \( \omega^2 \mu \epsilon \) hits an eigenvalue.

d) Suppose instead you had used the N-1 eigenfunction expansion method what would happen when \( \omega^2 \mu \epsilon \) hits an eigenvalue and then increases further?

Buzz Lighyear sez “To infinity and beyond.”